

# Linger Thermo Theory: Simplifying the Finding of the Entropy of Mediums with Applications that Span from Astrophysics to Human Lifespan

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**Abstract** — A new, simple, method is revealed to yield the entropy of mediums such as a black-hole, a photon gas, and a flexible phase. These mediums span the scale of application from astrophysics to the derivation of lifespan bounds for organisms. The offered scheme surfaced through linger thermo theory (LTT), a novel, powerful, and emergent theory of thermal physics that integrates thermodynamics with its recently discovered time-dual named linderdynamics. The origin of LTT can be traced to the Universal Cybernetics Duality Principle (UCDP), discovered in the late 1970s in Kalman Linear Quadratic Gaussian (LQG) ‘continuous’ control, with its first application being Matched Processors, a practical parallel processing methodology for ‘quantized’ control whose development was also motivated by biofeedback research on neural networks computational strategies. Although UCDP based entropies for the aforementioned mediums are now available, a simple scheme guiding as well as enlightening their derivation via the novel LTT has not yet been offered. The revelation of this scheme, a medium degrees of freedom (DoF) dependent one, is studied here. In a cybernetics application, the method is found to simplify the finding of the entropy of a flexible phase water medium, leading to theoretical lifespan bounds for humans that sensibly support the Harman free radical theory of aging. The offered scheme relates in one simple equation four basic physical quantities of a homogenous medium. They are: 1) its entropy; 2) its total mass-energy; 3) its temperature; and 4) its DoF number.

**Keywords** — cybernetics principle, neural networks, thermal physics, quantum space/time theory, thermodynamics, linderdynamics, dark mass/energy, big-data, black-hole, photon-gas, flexible-phase, degrees of freedom, free radical theory of aging, lifespan

## I. INTRODUCTION

Thermodynamics reveals via its 2<sup>nd</sup> law that with the passing of time a closed system or medium evolves from order to disorder, not in the opposite direction [1]. As a measure of order one may use, for instance, the number of binary digits (or bits) needed to represent the medium microstates. The microstates are defined as the set of possible configuration that basic *elements* of a medium may assume as they are distributed throughout a given number of energy levels while subject to total energy and number of elements constraints. This type of ‘sourced microstate behavior’ by the medium is

modeled in this paper by the *elements Boltzmann entropy (EBE)*  $\hat{S}$  in  $J/K$  physical units defined as:

$$\hat{S} = k \ln \hat{W} = \ln 2k \log_2 \hat{W} = \ln 2k \hat{H} \quad (1)$$

where: a)  $k$  is the Boltzmann constant in  $J/K$  physical units; b)  $\hat{W}$  is the total number of configurations of  $J$  medium elements in  $\ell$  energy states; c)  $\hat{H} = \log_2 \hat{W}$  is the thermo source entropy (TSE) of the medium in *thermo-bit* ‘mathematical’ units; d) the  $i$ -th energy state has  $g_i$  sublevels where  $i$  runs from 1 to  $\ell$ ; and e) the  $\hat{W}$  configurations are subject to a fixed number of elements and a fixed total energy.

For basic mediums their TSE expressions have already been derived such as an uncharged, nonrotating black-hole [2], a photon gas [3], and a flexible phase [4]-[5]. These mediums span the scale of applications from astrophysics to the derivation of theoretical lifespan bounds for organisms. For example, in this paper theoretical lifespan bounds will be studied for humans via the TSE of a flexible phase water medium since 99% of our molecules are of water [6]. To derive these bounds the TSE will be adjusted to reflect chemical reactions that alter the homogeneity of the water medium. As part of this illustrative example it will be found that the derived lifespan bounds sensibly give support to the Harman free radical theory of aging [7]-[9]. Although TSE expressions for the aforementioned three mediums are already available, a simple scheme guiding as well as enlightening their derivation has not yet been offered. This is the problem addressed here.

This paper reveals a simple universal equation for the TSE of a *homogeneous medium* that surfaced from the study of its degrees of freedom (DoF). SI units will be assumed throughout for all the physical variables.

The revealed universal DoF-TSE equation (UDTE) for homogeneous mediums is:

$$\hat{H} = \frac{\hat{S}}{\ln 2k} = \frac{Mc^2}{N_{DoF} \ln 2kT} = \frac{1}{2 \ln 2} \frac{Mc^2}{c_h kT} \approx 0.72J \quad (2)$$

where: a)  $E=Mc^2$  is the mass-energy of the medium with  $M$  being its mass in  $kg$  units and  $c$  the speed of light in  $m/s$  units; b)  $T$  is temperature in *Kelvin* units; c)  $N_{DoF}$  is the number of degrees of freedom where each degree is characterized by the kinetic thermal energy  $kT/2$ ; d)  $c_h=N_{DoF}/2$  is the DoF heat-capacity of the medium; e)  $c_h kT=N_{DoF}kT/2$  is the kinetic thermal energy contributed by all the degrees of freedom; and f)  $J$  is the number of medium elements, first named ‘thermotes’ in [5], with each contributing the same kinetic

thermal energy as that contributed by the  $N_{DoF}$  of the medium, i.e.,  $c_h kT$ .

The UDTE of (2) is a new, simple, scheme for finding the entropy of homogeneous mediums. This would be the case since given the medium's mass to temperature ratio  $M/T$ , all that remains for us to do is to find the number of degrees of freedom of the medium. For a black-hole, for instance, it is two, i.e.,  $N_{DoF}=2$ , since all motions are limited to the 2D event-horizon surface of the black-hole [2], while for a photon gas it is three, i.e.,  $N_{DoF}=3$ , since for photons only 3D translational motions are available. On the other hand, for a flexible phase medium  $N_{DoF}$  exceeds the value of three since in addition to 3D translations, rotational and vibrational motions are also possible. Moreover, when comparing these TSE results it is further noted that the number of thermo-bits for a black-hole is at least 50% greater than that of the two other mediums.

A further advantage of the UDTE is that it also reveals an appealing *synergistic thermal/non-thermal duality* for the description of the masses and energies of mediums. Firstly, *non-thermal masses and energies* are defined for either basic particles, such as fermions or bosons [10], or composite particles, e.g., a diatomic water molecule with a mass  $m$  of  $3 \times 10^{-26}$  kg and a mass-energy of  $e=mc^2=27 \times 10^{-10}$  Joules. Secondly, basic *thermal energies and masses* are defined for the medium elements that give rise to the UDTE of (2). These medium elements are thermotes that were defined earlier for (2) as the kinetic thermal energy  $e_T=c_h kT$  contributed by all the degrees of freedom of the medium with its corresponding energy-mass given by  $m_T=e_T/c^2$ . The energy contributed by the thermotes can then be used to generate the non-thermal mass-energy of either a particle or an entire medium. Using this approach one finds that  $j$  thermotes generate the mass-energy of a particle as follow:

$$j=e/e_T=mc^2/c_h kT, \quad (3)$$

while  $J$  thermotes generate the mass-energy  $E=Mc^2$  of an entire medium as follow:

$$J=E/e_T=Mc^2/c_h kT. \quad (4)$$

As an illustration, when  $M=70$  kg,  $T=310$  K,  $c_h=2.341$  and  $m=3 \times 10^{-26}$  kg one assigns  $j=2.691 \times 10^{11}$  thermotes to each water molecule and  $J=6.279 \times 10^{38}$  thermotes to the entire medium. Using this number of thermotes definition it then follows that the number of thermo-bits of a homogeneous medium given by  $\hat{H}$  in (2) is 72% of the number of medium thermotes  $J$ . In this way one concludes that to produce a single thermo-bit the medium uses 72% of the energy of a single thermote, or inversely, that one thermote of a medium enables the production of  $2 \ln 2 = 1.39$  thermo-bits.

The UDTE of (2) and the just described *non-thermal/thermal duality* for the description of the masses and energies of mediums is anchored in a nascent linger thermo theory (LTT) of thermal physics. This theory has cybernetics roots [11]. More specifically, it inherently surfaced from the Universal Cybernetics Duality Principle (UCDP) that was discovered in the late-1970s in Kalman Linear Quadratic Gaussian (LQG) 'continuous' control [12]-[13] as part of graduate studies in three cybernetics research areas, i.e., communications, biofeedback and optimum control [14]-[17].

The LTT is reviewed in Section II where its UCDP roots are highlighted.

The organization of this paper is as follows. In Part II a review of LTT is advanced that includes its UCDP roots as well as its contributions to astrophysics and thermal physics. In Part III the TSE of three basic homogeneous mediums are simply derived via the UDTE of (2). In Part IV sensible LTT based theoretical lifespan bounds for individuals are derived that make use of the UDTE and are found to strongly support the Harman free radical theory of aging. The paper ends with conclusions that include the timeline of development of the UCDP for ease of recall in applications.

## II. LINGER THERMO THEORY

The epicenter of LTT where the UDTE is anchored is found in cybernetics [11], more specifically in the universal cybernetics duality principle or UCDP. Universal cybernetics studies control and communication in living and non-living systems. The UCDP separates at each processing stage the solution of a cybernetics problem into a '*deterministic or certainty*' continuous and/or quantized state control problem, and a '*stochastic or uncertainty*' continuous (estimation) and/or quantized (detection) state communication problem. While at each observation stage the control problem assumes a certainty spacetime for the future, the communication problem assumes an uncertainty spacetime for the past [4]. Such separation, is further linked to cybernetic strategies that use mathematical and physical dualities to inherently lead to efficient (or compressed) as well as affordable solutions. Moreover, the UCDP has a *duality language and terminology* that is both *unique and enabling* since it has naturally led us to novel and sensible scientific theories such as the LTT. As mentioned earlier the UCDP was first identified in the late-1970s in Kalman LQG 'continuous' control [12]-[13] as part of graduate studies in cybernetics [14]-[15] and [17]. The first application of the UCDP was Stochastic Matched Processors (SMP), a practical parallel processing methodology for 'quantized' control whose development was significantly motivated by research on biofeedback research on neural networks computational strategies [16]. In the SMP scheme 'quantized control' actions are taken via a suitably small number of *certainty present to future* based Matched Processors [14], [17], while 'quantized state' detections are executed by *uncertainty past to present* based Matched Filters [18].

After the discovery of SMP in the late-1970s it wasn't until the mid-2000s when major transformational UCDP driven scientific methodologies were successively revealed. We next review these methods in several subsections. Firstly Latency Information Theory (LIT) is reviewed in Subsection A, which surfaced when a novel Latency Theory [19] was discovered to be the *certainty present to future* based dual of the *uncertainty past to present* based Information Theory [20]. Secondly Motion Retention Theory (MRT) is reviewed in Subsection B which surfaced when a novel Laws of Retention in Physics was discovered to be the *uncertainty past to present* based dual of the *certainty present to future* based Laws of Motion in Physics [21]. Thirdly, Linger Thermo Theory (LTT) is reviewed in

Subsection C which surfaced when a novel Lingerdynamics was found to be the *certainty present to future* based dual of the *uncertainty past to present* based Thermodynamics [22]-[23].

#### A. The Birth of LIT in the Mid-2000s

The birth of LIT was motivated by a Defense Advanced Research Projects Agency (DARPA) University Grant [25] for research on high-performance radar [26]-[27] that used SAR images of the earth to pointedly improve target detections. These SAR images as well as their infrastructural requirements, called here big-data, needed to be compressed. However, soon it was found that to achieve high radar performance with highly *space-compressed sourced data* it was also necessary to *time-compress the radar processor* in a synergistic space-time manner [26]. This synergistic solution was found in a novel Power Centroid Radar scheme that evolved naturally from LIT which integrates Information Theory [20] with its time dual, i.e., Latency Theory [19]. LIT was the second major application of the UCDP after SMP and studies the space and time resources of systems for their efficient and affordable representation. This is done through two entropic space metrics and two ectropic time metrics (ectropy is the time dual of entropy) that are time-invariant or stationary and are briefly described next from a global perspective.

One of two entropic space metrics of LIT is the information source entropy (ISE) in information bit (*or info-bit*) math units, defined in C.1.1.2 with symbol  $H$  (it is the same as the Shannon source entropy [20]). The ISE is used in information theory to guide the design of a *channel and source integrated (CSI) coder*. The CSI coder communicates  $H$  info-bits through a channel subject to the *channel-capacity* metric  $C=(H-H_L)/H$ , where  $H_L (\leq H)$  is viewed as a channel penalty paid for its use. The second novel entropic space metric of LIT is the ‘surface area space’ information retainer entropy (IRE) in information square meter (*or info-m<sup>2</sup>*) physical units, defined in C.1.2.2 with symbol  $N$ . The IRE is used in our enhanced information theory to guide the design of a *sensor and retainer integrated (SRI) coder*. The SRI coder observes  $N$  info-m<sup>2</sup> across a sensor subject to the sensor-scope metric  $I=(N-N_L)/N$ , where  $N_L (\leq N)$  is viewed as a sensor penalty paid for its use.

On the other hand, one of two ectropic time metrics of LIT is the ‘processing steps’ information processor ectropy (IPE) in latency binary operator (*or late-bor*) math units, defined in C.1.3.2 with symbol  $K$ . The IPE is used in latency theory to guide the design of a *sensor and processor integrated (SPI) coder*. The SPI coder observes  $K$  late-bors across a sensor subject to the novel processor-consciousness metric  $F=(K-K_L)/K$ , where  $K_L (\leq K)$  is viewed as a sensor penalty paid for its use. The second ectropic time metric of LIT is the ‘delay time’ information mover ectropy (IME) in latency second (*or late-sec*) physical units, defined in C.1.4.2 with symbol  $A$ . The IME is used in our enhanced latency theory to guide the design of a *channel and mover integrated (CMI) coder*. The CMI coder communicates  $A$  info-sec through a channel subject to the channel-stay metric  $T=(A-A_L)/A$ , where  $A_L (\leq A)$  is viewed as a channel penalty

paid for its use.

The aforementioned LIT methodology inherently leads to affordable and efficient system solutions not only in radar [27] but in numerous others multi-dimensional data communication and observation applications, e.g., in tomography problems for the health sciences [28] and in image processing. Before LIT other powerful analysis and design tools were discovered through information theory. These tools all made use of *coupled eigensystem and Wiener-Hopf equations* first derived three decades ago (see [29] and [30] for conference and journal versions, respectively). These universal equations for the design of *coupled prediction and transformation models for systems*, yielded a *MMSE predictive-transform (PT) source model* that inherently provided a transformation ‘compression’ mechanism for the state of a system. The MMSE PT source model was first applied to source coding in [29]-[30] and then to the unification of Kalman and Wiener estimation under the name *Predictive Transform Estimation* (see [31] for its first journal publication). In [32]-[34] the universality of the MMSE PT source modeling approach was further demonstrated with applications to detection, control, channel and source integrated or CSI coding, including USA patents.

#### B. The Birth of MRT in the Late-2000s

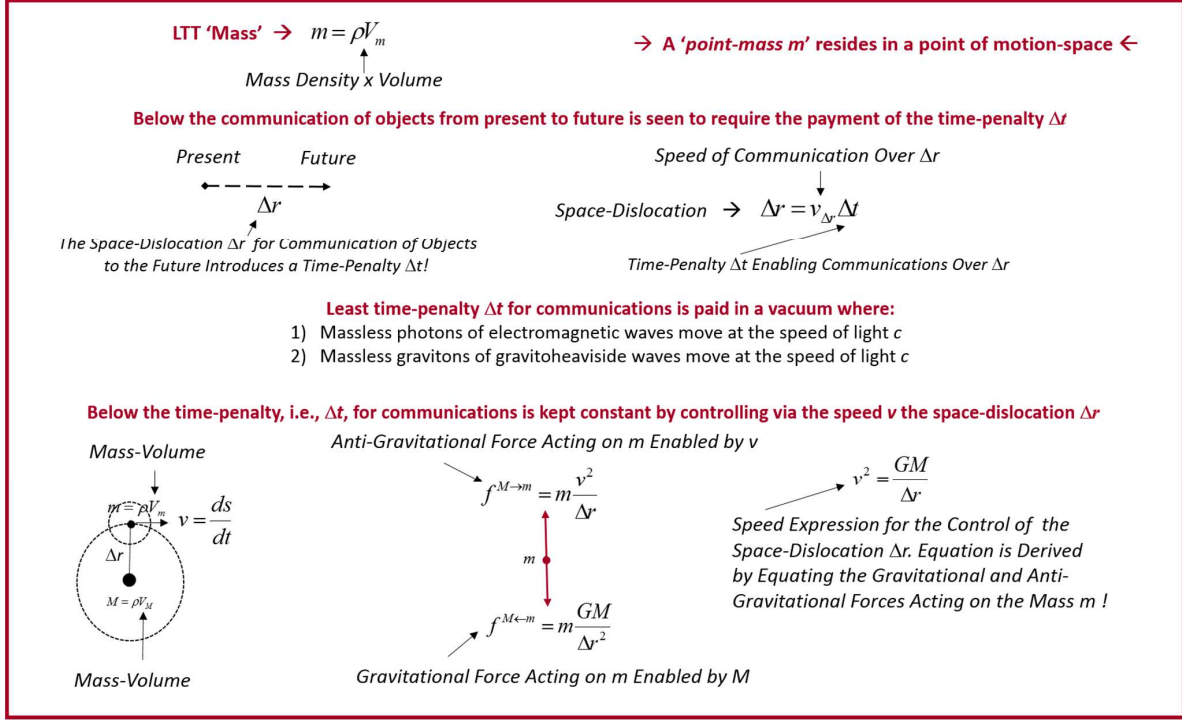
Like SMP and LIT the birth of MRT was revealed through the UCDP [4]. This revelation started when the Laws of Retention in Physics were revealed [21] as the *uncertainty past to present* dual of the *certainty present to future* Laws of Motion in Physics. These duality laws formed together a novel MRT which is characterized by physical and mathematical dualities. In the next three subsections these dualities are reviewed.

**B.1 The vacuum/black-hole physical-duality:** Highlighted in the MRT was that a vacuum and a black-hole formed together a *vacuum/black-hole physical-duality*. In this physical-duality framework a vacuum offered the least resistance to the motion of mass/energy, while a black-hole offered the least resistance to the retention of mass/energy.

**B.2 The speed-equation/pace-equation math-duality:** The efficiency of the vacuum/black-hole physical duality was in turn measured by a mathematical duality consisting of a *speed-equation and its space-dual, i.e., a pace-equation*. *Speed* with symbol  $v$  was defined as the mathematical ratio  $d/t$  where  $d$  is the amount of space-dislocation (or motion displacement) of mass/energy and  $t$  is the amount of time-penalty (or delay) suffered to achieve  $d$ . On the other hand, the novel term ‘*pace*’ with symbol  $\Pi$  was defined as the math ratio of  $\tau$  over  $V$ :

$$\Pi = \tau / V \quad (5)$$

where  $\tau$  is the amount of time-dislocation (or retention duration) of mass/energy and  $V$  is the amount of space-penalty (or storage) suffered to achieve  $\tau$ . Via this speed/pace duality it was then found that while the speed of light  $c$  offered the maximum possible speed in a vacuum, the pace of dark (dark is the black-hole dual of light in a vacuum) with symbol  $\chi$  offered the maximum pace possible in a black-hole. In [21] an expression



**Fig.1. Control of the gravity induced communication distance between two masses.**

was first found for  $\chi$  as a function of the speed of light  $c$ , the reduced Planck constant  $\hbar$  and the gravitational constant  $G$  according to (see Appendix A for proof):

$$\chi = \frac{480c^2}{G\hbar} = 6.1203 \times 10^{63} \text{ sec} / m^3 \quad (6)$$

$$\chi = 480c^2 / G\hbar$$

**B.3 The gravity-equation/gravidness-equation math-duality:** This MRT duality has the gravitational force equation as one of its two pillars, also seen on the bottom of Fig. 1. In it the attraction force exerted by a point-mass  $M$  on the point-mass  $m$  (or in the opposite direction, i.e., by  $m$  on  $M$ ) is modeled by:

$$f^{M \leftarrow m} = GMm / \Delta r^2 \quad (7)$$

where  $G$  is the gravitational constant and  $\Delta r$  is the space-dislocation (or communication distance) between  $m$  and  $M$ . In this figure it is shown how the distance  $\Delta r$  can be controlled by an *anti-gravitational force* that is *speed  $v$  enabled* according to:

$$f^{M \rightarrow m} = mv^2 / \Delta r \quad (8)$$

$$v = \sqrt{GM / \Delta r} = \omega \Delta r = 2\pi \Delta r / T = \pi \Delta r / \hat{A} \quad (9)$$

where: a)  $v$  in (9) is derived by equating (7) and (8) and denotes the speed of  $m$  with respect to  $M$  that keeps  $\Delta r$  fixed in value; b)  $\omega$  is angular velocity; c)  $T$  is the period of one revolution centered about  $M$ ; and d)  $\hat{A} = T / 2$  is one half of  $T$  that is later used to define one of four LTT metrics.

Finally it is also shown on the top half of Fig. 1 how to have 'communications' over  $\Delta r$  meters in either direction,

i.e., from  $m$  to  $M$  or from  $M$  to  $m$ , one must first suffer a time-penalty of  $\Delta t$  secs where all communications start in the present and end in the future. This time-penalty is also noted to be the smallest in a *vacuum* where *massless photons of electromagnetic waves* and *massless gravitons of gravitoheavise waves* move at the speed of light  $c$  [21].

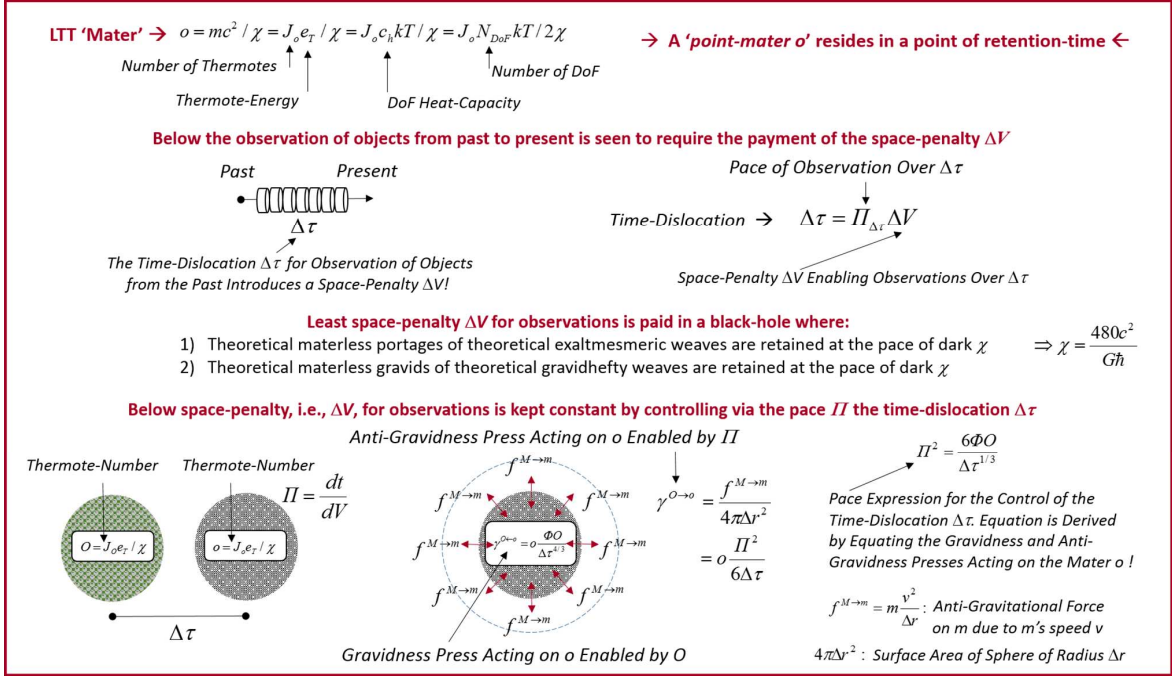
The LTT gravidness press equation is the second pillar of the gravitational/gravidness duality, also seen on the bottom of Fig. 2. In it the attraction press (in  $Pa$  pressure units) exerted by a point-mater  $O$  (an *energy per pace* physical quantity) on the point-mater  $o$  (or in the reverse direction, i.e., by  $o$  on  $O$ ) is modeled by:

$$\gamma^{O \leftarrow o} = \Phi O o / \Delta \tau^{4/3} = f^{M \leftarrow m} / 4\pi \Delta r^2 = f^{M \leftarrow m} / \hat{N} \quad (10)$$

where  $\Phi$  is the gravidness constant and  $\Delta \tau$  is the time-dislocation (or observation delay) between  $o$  and  $O$ .  $\gamma^{O \leftarrow o}$  is the attraction press or pressure exerted by the uniformly distributed attraction force  $f^{M \leftarrow m}$  of (8) on the surface of a spherical medium with the surface area expressed as  $\hat{N} = 4\pi \Delta r^2$ , see bottom of Fig. 2. This surface area is later used to define the second of four LTT metrics. An equation for  $\Phi$  is also available and was first derived in [21] using black-hole conditions (see Appendix B for proof). It relates  $\Phi$  to the  $G$ ,  $c$  and  $\chi$  in a 'fundamental' way as follows:

$$\Phi = \sqrt[3]{4\pi\chi^{10} / 81c^{12}G} = 1.8619 \times 10^{168} \text{ Pa} \cdot \text{sec}^{22/3} \cdot \text{N}^{-2} \cdot \text{m}^{-8} \quad (11)$$

Also under black-hole conditions 'mater' (the retention dual of mass) can be expressed as the ratio of mass-energy and



**Fig. 2. Control of the gravidness induced observation delay between two maters.**

the pace of dark  $\chi$ , first derived in [21], according to:

$$o = mc^2 / \chi \quad (12)$$

where: a) (12) relates the mater  $o$  to the mass  $m$  in black-hole conditions; and b) the product of  $o$  and  $\chi$ , i.e.,  $o\chi$ , is called endurance or *dark-energy* when in a black-hole and  $m$  is called *dark-mass* when its density is that of a black-hole. On top of Fig. 2 it is also seen how mater can be expressed in terms of thermotes according to:

$$o = mc^2 / \chi = J_o e_T / \chi \quad (13)$$

where  $J_o$  is the numbers of thermotes in the medium and  $e_T = c_h kT$  is the thermote energy where  $c_h = N_{DoF}/2$  is the medium's DoF heat-capacity. In Fig. 2 it is shown how the amount of delay  $\Delta\tau$  between  $o$  and  $O$  can be controlled by an *anti-gravidness press* that is *pace  $\Pi$*  enabled according to (with a mathematical structure similar to that of the anti-gravity force of (8)):

$$\gamma^{o \leftarrow O} = o\Pi^2 / 6\Delta\tau \quad (14)$$

$$\Pi = \sqrt{\frac{6\Phi O}{\Delta\tau^{1/3}}} = \frac{16\Delta r}{V} \sqrt{\frac{5\pi M \Delta\tau}{\hbar}} = \frac{48}{\hat{N}} \sqrt{\frac{5\pi M \Delta\tau}{\hbar}} = \frac{12}{\Delta r^2} \sqrt{\frac{10M}{\hbar}} \Delta\tau \quad (15)$$

where: a) the  $\Pi$  in (15) is derived by equating (10) and (14) and denotes the pace of  $o$  with respect to  $O$  that keeps  $\Delta\tau$  fixed in value; b)  $\hbar$  is the Planck constant; and c)  $V$  is the volume of the medium assumed spherical with radius  $\Delta r$ , and is related to  $\hat{N}$  according to  $\hat{N} = 4\pi\Delta r^2 = 3V / \Delta r$ .

Finally it is also shown on the top half of Fig. 2 how to have 'observations' over  $\Delta\tau$  seconds in either direction, i.e., from  $o$  to  $O$  or from  $O$  to  $o$ , one must first suffer a space-

penalty of  $\Delta V$  cubic meters where all observation start in the past and ends in the present. This space-penalty is also noted to be the smallest in a *black-hole* where *theoretical materless portages* (the retention dual of a photon) of *theoretical exaltmesmeric weaves* (the retention dual of electromagnetic waves) and *theoretical materless gravids* (the retention dual of gravitons) of *theoretical gravidhefty weaves* (the retention dual of gravitoheaviside waves) are retained at the pace of dark  $\chi$ . These MRT dualities were first proposed in [21].

### C. The Birth of LTT in the Late-2000s

Like SMP, LIT and MRT the birth of LTT was revealed through the UCDP [4]. This revelation started when Lingerdynamics was identified as the *certainty present to future* dual of the *uncertainty past to present* Thermodynamics [22]-[23]. These duality laws form together a novel LTT which is reviewed in the next three subsections. First in C.1 LTT metrics are defined and then in C.2 the universal linger thermo equation (ULTE) which brings together all the LTT metrics. The ULTE will serve as the launching pad for the theoretical lifespan bounds of Section IV. Finally in C.3 linger thermo engines are treated (*the reading of C.3 may be at first skipped since it is not directly connected to our UDTE results*).

#### C.1. The Four LTT Metrics

We first consider two LTT thermodynamics space-metrics and then the last two LTT lingerdynamics time-metrics.

**C.1.1. The TSE Metric:** The first of the two time-varying entropic space-metrics of LTT is the TSE  $\hat{H}$  in thermo-bit units given by [35]:

$$\hat{H} = E[T_S(X)] = \sum_{i=1}^{\hat{W}} \log_2 \frac{1}{p(X_i)} \Big|_{p(X_i)=1/\hat{W}} = \log_2 \hat{W} = \frac{\hat{S}}{k \ln 2} \quad (16)$$

where  $E[T_S(X)]$  expresses the expectation of a set of  $\hat{W}$  values  $\{T_S(X_i)\}$  with each  $T_S(X_i)$  value defined as:

$$T_S(X_i) = \log_2(1/p(X_i)) \quad \text{for } i=1, \dots, \hat{W} \quad (17)$$

The  $T_S(X_i)$  expression (17) denotes the minimum amount of sourced thermo-bits that can be used to describe the sourcing of  $X_i$ , where  $X_i$  is one of  $\hat{W}$  possible realizations of the random microstate  $X$  and  $p(X_i)$  is the probability of sourcing  $X_i$ . In particular,  $X_i$  denotes one possible distribution of  $J$  medium elements in  $\ell$  energy states, with the  $i$ -th energy state containing  $g_i$  possible sublevels for all  $i$ . In LTT the medium elements are its  $J$  thermotes (4), organized in groups of  $j$  thermotes for each medium particle according to (3). In particular, when the  $\hat{W}$  sourced realizations or configurations of  $X$  are equally likely in occurrence, i.e.,  $p(X_i)=1/\hat{W}$  for all  $i$ , a maximum value for  $E[T_S(X)]$  results given by  $\log_2 \hat{W}$  which leads, in turn, to the shorter version of (16) given by:

$$\hat{H} = \log_2 \hat{W} = \hat{S} / \ln 2k \quad (18)$$

The  $\hat{W}$  possible configurations of  $X$  require, however, that each be constrained to a fixed number of thermotes  $J$  whose energy is the same as the fixed total medium energy  $Mc^2$ . The maximization of  $\hat{W}$  subject to these two constraints, i.e., a fixed number of medium thermotes and a fixed amount for their energy, yields for *composite* particles, such as the diatomic  $H_2O$ , the Boltzmann distribution for the occupancy of the  $\ell$  energy states. On the other hand, for *fundamental* fermion and boson particles the Fermi-Dirac and Bose-Einstein distributions can be derived, respectively, while subject to the aforementioned constraints. When these distributions do not change with time, the medium is said to be in *thermal equilibrium* since the medium temperature does not change.

**C.1.1.1. The 2<sup>nd</sup> Law of Source Thermodynamics.** Linked to the TSE of (16) one then finds *the 2<sup>nd</sup> law of source thermodynamics* (similar to the 2<sup>nd</sup> law of thermodynamics) which states that for a closed medium the TSE will continuously increase with time. This law reflects an increase in the number of microstate configurations or thermo-bits, described here as the evolution of the medium from order to disorder [1]. Moreover, this *source-entropic evolution towards disorder* conveys an ‘aging’ or ‘arrow of time’ for the medium of an *order nature*.

**C.1.1.2. The Information Source Entropy or ISE:** It is now noted that the time-invariant version of (16) is the LIT Shannon source entropy  $H$  or ISE given by [20]:

$$H = E[I_S(X')] = \sum_{i=1}^{\Omega} I_S(X'_i) p(X'_i) \quad (19)$$

$$I_S(X'_i) = \log_2 \frac{1}{p(X'_i)} \quad \text{for } i=1, \dots, \Omega \quad (20)$$

where  $H$  is the expected value of information binary digits or info-bits  $I_S(X')$  assigned to the discrete random output or outcome  $X'$  of an *information-source* according to (20) for  $\Omega$  possible realizations. An information-source example is an elementary image-source [29]-[34] that sources into a single

picture element (or pixel) one of  $\Omega$  possible levels  $\{X'_i\}$  whose *binary digits information amount* is bounded by (20).

**C.1.2. The TRE Metric:** The second of two time-varying entropic space-metrics of LTT is the thermo retainer entropy (TRE)  $\hat{N}$  in *thermo- $m^2$*  units given by [35]:

$$\hat{N} = E[T_R(X)] = \sum_{i=1}^{\hat{W}} 4\pi r_i^2 p(X_i) \Big|_{p(X_i)=1/\hat{W}, r_i=r} = 4\pi r^2 = \frac{3V}{r} \quad (21)$$

where  $E[T_R(X)]$  expresses the expectation of a set of  $\hat{W}$  values  $\{T_R(X_i)\}$  with each  $T_R(X_i)$  value defined as:

$$T_R(X_i) = 4\pi r_i^2 \quad \text{for } i=1, \dots, \hat{W} \quad (22)$$

The  $T_R(X_i)$  expression (22) denotes the minimum amount of ‘surface area’ in thermo- $m^2$  units that can be used to describe the retention of the microstate  $X_i$  where  $4\pi r_i^2$ ,  $V_i = 4\pi r_i^3 / 3$  and  $r_i$  denote a sphere’s surface area, volume, and radius, respectively. Among all the possible closed medium shapes, a spherical medium offers the least amount of surface area coverage for a given volume. As is the case for the TSE, the  $\hat{W}$  microstates of the TRE are assumed to be both equally likely and their retention volumes identical in value which leads, in turn, to the shorter version of (21) given by:

$$\hat{N} = 4\pi r^2 = 3V / r \quad (23)$$

Also, through their common  $\hat{W}$  thermal microstates, the TSE and TRE can be implicitly related as follows:

$$\hat{N} = 4\pi r^2 = 3V / r = 3\tau / r\Pi \quad (24)$$

where  $\tau$  is defined as the duration of the retention of some (or all) of the  $\hat{H}$  thermo-bits in the medium volume, and

$$\Pi = \tau / V \quad \text{in } \text{sec}/m^3 \text{ units} \quad (25)$$

defines the pace of this retention (pace in *sec/m<sup>3</sup>* units is the retention dual of speed [21]).

**C.1.2.1. The 2<sup>nd</sup> Law of Retainer Thermodynamics:** Linked to the TRE of (21) one then finds *the 2<sup>nd</sup> law of retainer thermodynamics* which states that for a closed medium the TRE will continuously increase with time. This law is due to an increase in the surface area of enclosure, which is described here as the evolution of the medium surface area from retention to release. Moreover, this *retainer-entropic evolution towards release* conveys an ‘aging’ or ‘arrow of time’ for the medium of a *retention nature*.

It is of interest to note that the 2<sup>nd</sup> law of retainer thermodynamics together with the UDTE of (2) informs us that for a closed homogeneous medium its *thermal equilibrium* must be continuously changing since the medium temperature is continuously decreasing in value.

**C.1.2.2. The Information Retainer Entropy or IRE:** It is now noted that the time-invariant version of (21) is the LIT information retainer entropy  $N$  or IRE given by [4]:

$$N = E[I_R(X')] = \sum_{i=1}^{\Omega} I_R(X'_i) p(X'_i) \quad (26)$$

$$I_R(X'_i) = 4\pi r_i^2 \quad \text{for } i=1, \dots, \Omega \quad (27)$$

where ‘ $N$ ’ is the expected value of information square meters or *info- $m^2$*   $I_R(X')$  assigned to the discrete random output or outcome  $X'$  of an *information-retainer* according to (27) for  $\Omega$

possible realizations. An information-retainer example is a cargo container that retains in each storage container one of  $\Omega$  possible objects  $\{X_i\}$  whose volume's *surface area information amount* is bounded by (27).

**C.1.2.3. The 2<sup>nd</sup> Law of Retainer Thermodynamics and the UDTE:** In the context of the TRE  $\hat{N}$  of (21) and its 2<sup>nd</sup> law a further investigation of (2) reveals that for a closed homogenous-medium with  $Mc^2$  and  $N_{DoF}$  fixed in value, the temperature of the medium decreases as the TSE  $\hat{H}$  of (16) increases with time due to the 2<sup>nd</sup> law of source thermodynamics. The decrease of this temperature is noted to be in perfect accord with the 2<sup>nd</sup> law of retainer thermodynamics. This retainer law, which is consistent with the observed expansion of our Universe [24], states that the surface area enclosing the medium increases, or its mass density decreases, with time which, in turn, results in the temperature of the medium decreasing as the TSE increases. Thus it can be stated here that the UDTE advanced in this paper is both synergistically and sensibly supported by the 2<sup>nd</sup> laws of source and retainer thermodynamics of LTT.

**C.1.3. The TPE Metric:** The first of two time-varying ectropic time-metrics of LTT is the thermo processor ectropy (TPE)  $\hat{K}$  in linger binary operator (linger-bor) units given by [35]:

$$\hat{K} = \max[L_P(h_i) = \log_{C(h_i)} h_i : i = 1, \dots, \Lambda] \Big|_{h_i \rightarrow \hat{H}} = \log_{C(\hat{H})} \hat{H} = \sqrt{\hat{H}} \quad (28)$$

where  $\max[L_P(h_i):1, \dots, \Lambda]$  expresses the maximum value in a set of  $\Lambda$  values  $\{L_P(h_i)\}$  with each  $L_P(h_i)$  value defined as:

$$L_P(h_i) = \log_{C(h_i)} h_i \text{ for } i=1, \dots, \Lambda \quad (29)$$

The  $L_P(h_i)$  of (29) denotes the minimum amount of processing linger-bors that can be used to derive from  $h_i$  thermo-bits the  $i$ -th processor output, where  $C(h_i)$  is a constraint on the maximum number of inputs that basic logic operators can have when operating on  $h_i$ . The  $C(h_i)$  constraint which affects the amount of linger-bors for  $h_i$  processing according to (29), is the ‘certainty’ processing dual of the ‘uncertainty’ probability  $p(X_i)$  that affects the amount of thermo-bits of a sourced  $X_i$  according to (20). For instance, if  $\Lambda=2$  outputs with  $h_1=8$  thermo-bit inputs,  $h_2=4$  thermo-bit inputs and  $C(h_1)=C(h_2)=2$ , one then finds  $L_P(h_1)=3$  linger-bors,  $L_P(h_2)=2$  linger-bors and  $\hat{K} = \max[3,2]=3$  linger-bors. As another example taken from ‘time-invariant’ digital design [37], consider a one-bit full adder with  $\Lambda=2$  outputs (for the sum and carry-out),  $h_1=h_2=3$  info-bits inputs (for the  $a$ ,  $b$  and carry-in info-bit inputs) and  $C(h_1)=C(h_2)=3$  (e.g., when wired logic with three-input open-collector NAND gates are used) one then finds  $L_P(h_1)=L_P(h_2)=1$  late-bor and  $\hat{K} = \max[1,1]=1$  late-bor (not included is an additional late-bor that may be needed to derive inverted info-bit inputs). The set  $\{L_P(h_i)\}$  thus characterizes via linger binary operators the processing of the set of thermo-bits  $\{h_1, \dots, h_\Lambda\}$  where each group of  $h_i$  thermo-bits is assumed processed via logical binary operators constrained to  $C(h_i)$  inputs. Moreover, when the processing is performed on all the thermo-bits of the TSE, i.e.,  $h_i = \hat{H}$  for all  $i$  and  $C(\hat{H})$  approaches the value of 1, this leads, in turn, to the shorter version of (28) given by:

$$\hat{K} = \log_{C(\hat{H})} \hat{H} = \sqrt{\hat{H}} \quad (30)$$

where it is assumed that  $\hat{H}$  is a large number. It is of interest to note here that as  $C(h_i)$  approaches one for all  $i$ ,  $\hat{K}$  attains its largest possible value. Thus the constraint condition  $C(h_i) \rightarrow 1$  maximizing  $\hat{K}$  is said to be the *certainty* dual of the uncertainty probability condition  $P(X_i) \rightarrow 1/\hat{W}$  maximizing the TSE  $\hat{H}$ .

**C.1.3.1. The 2<sup>nd</sup> Law of Processor Lingerdynamics:** Linked to the TPE of (28) one then finds the 2<sup>nd</sup> law of processor lingerdynamics stating that for a closed medium the TPE will continuously increase with time. This law is due to an increase in the levels of processing, described here as the evolution of the levels of processing of the medium from connection to disconnection. Moreover, this *processor-ectropic evolution towards disconnection* conveys an ‘aging’ or ‘arrow of time’ for the medium of a *connection nature*. It is further noted that the processor 2<sup>nd</sup> law is supported by (30) which has the TPE  $\hat{K}$  increasing as the TSE  $\hat{H}$  increases in value.

**C.1.3.2. The Information Processor Ectropy or IPE:** It is now noted that the time-invariant version of (28) is the LIT information processor ectropy  $K$  or IPE given by:

$$K = \max[L_P(h_i') : i = 1, \dots, \Lambda] \quad (31)$$

$$L_P(h_i') = \log_{C(h_i')} h_i' \text{ for } i=1, \dots, \Lambda \quad (32)$$

where  $K$  denotes the maximum value in a set of  $\Lambda$  latency binary operators or *late-bors* values  $\{L_P(h_i')\}$  assigned to the  $\Lambda$ -dimensional vector output of an *information-processor* according to (32) with  $h_i'$  denoting the number of info-bits processed to yield the  $i$ -th output and  $C(h_i')$  is an upper constraint in the number of inputs that basic logic operators operating on  $h_i'$  may have. An information-processor example is a full-adder (discussed earlier) with  $\Lambda$  outputs, where each output results from the processing of one of  $\Lambda$  vectors of info-bits  $\{h_i'\}$  subject to the logic operator constraints  $\{C(h_i')\}$ , with its *latency levels* bounded by (32).

**C.1.4 The TME Metric:** The second of the two time-varying ectropy time-metrics of LTT is the thermo mover ectropy (TME) with symbol  $\hat{A}$  in *linger-sec* units given by [35]:

$$\hat{A} = \max[L_M(h_i) = \frac{\pi r_i}{v_i} : i = 1, \dots, \Lambda] \Big|_{r_i=r, v_i=v} = \frac{\pi r}{v} = \sqrt{\frac{\pi \hat{N}}{4v^2}} = \frac{\pi^{1/4} \hat{N}^{3/4}}{\sqrt{8GM}} \quad (33)$$

where  $\max[L_M(h_i):1, \dots, \Lambda]$  expresses the maximum value in a set of  $\Lambda$  values  $\{L_M(h_i)\}$  with each  $L_M(h_i)$  value defined as:

$$L_M(h_i) = \pi r_i / v_i \text{ for all } i \quad (34)$$

The  $L_M(h_i)$  expression (34) denotes the minimum amount of linger-secs derived by a test point-mass  $m_i$  that revolves at a constant speed  $v_i$  about a point in 2D space. This space-dislocation of the point-mass is such that the enclosed area for each rotation  $R$  is fixed in value and only  $1/2$  of the displacement around the point is noted. The shape of the fixed enclosed area  $R$  yielding the least time delay (34) is that of a circle of radius  $r_i$ , a  $1/2$  circumference of  $\pi r_i$ , and a fixed enclosed area of  $\pi r_i^2$ . Moreover, when one substitutes  $r$  for  $r_i$  and  $v$  for  $v_i$  in (34) for all  $i$  one derives:

$$\hat{A} = \pi r / v = \sqrt{\pi \hat{N} / 4v^2} \quad (35)$$

where  $\hat{N} = 4\pi r^2$ , which is noted to be part of (33). Moreover, if the center of the circle giving rise to (35) is at the same location where the center of mass of a spherical medium of radius  $r$  resides, one then finds that:

$$v = \sqrt{GM/r} \quad (36)$$

where: a)  $M$  is the point-mass of the medium that is found at its center of mass; b)  $G$  is the gravitational constant; and c)  $v$  is a perpetual centripetal speed of  $m_t$  revolving around the point-mass  $M$ . The relation (36) is arrived at by equating the gravitational force  $f^{M \leftarrow m_t} = Gm_t M/r^2$  of the point-mass  $M$  of the medium on the test point-mass  $m_t$  and the anti-gravitational force from  $m_t$  given by  $f^{m_t \leftarrow M} = m_t v^2/r$ . From (36) one notes that when the medium mass  $M$  is time invariant,  $v$  would decrease in value as the spherical radius of the medium  $r$  increases.

Next using (36) in (35) one then arrives at the shorter version of (33) given by:

$$\hat{A} = \pi r / v = \sqrt{\pi \hat{N} / 4v^2} = \pi^{1/4} \hat{N}^{3/4} / \sqrt{8GM} \quad (37)$$

**C.1.4.1. The 2<sup>nd</sup> Law of Mover Lingerdynamics:** Linked to the TME of (33) one then finds the 2<sup>nd</sup> law of mover lingerdynamics stating that for a closed medium the TME will continuously increase with time. This law is due to an increase in the period of rotational motion, i.e.,  $2\hat{A}$ , which is described here as the evolution of the medium from mobility to immobility. Moreover, this mover-ectropic evolution towards immobility conveys an ‘aging’ or ‘arrow of time’ for the medium of a mobility nature. It is further noted that the mover 2<sup>nd</sup> law is supported by (37) which has the TME  $\hat{A}$  increasing as the TRE  $\hat{N}$  increases in value.

**C.1.4.2. The Information Mover Ectropy or IME:** It is now noted that the time-invariant version of (33) is the LIT information mover ectropy  $A$  or IME given by:

$$A = \max \{L_M(h_i') : i = 1, \dots, \Lambda\} \quad (38)$$

$$L_M(h_i') = d_i / v_i \quad \text{for } i=1, \dots, \Lambda \quad (39)$$

where ‘ $A$ ’ denotes the maximum value in a set of  $\Lambda$  latency seconds or late-secs values  $\{L_M(h_i')\}$  assigned to the  $\Lambda$ -dimensional vector output of an information-mover according to (39) with  $d_i$  denoting the space-dislocation of  $h_i'$  info-bits of an object moving at the fastest possible speed of  $v_i$ . An information-mover example is a race track system with  $\Lambda$  outputs, where each output, consisting of a runner crossing the finish line, results from the motion from a starting location to a finishing one of an assigned runner that carries one of  $\Lambda$  info-bit vectors  $\{h_i'\}$  with his latency delay bounded by (39).

## C.2. The Universal Linger Thermo Equation (ULTE)

The definitions of the entropy and ectropy metrics given in the previous subsection give rise to a universal linger thermo equation (ULTE) that inherently relates them. This ULTE, in turn, has resulted in the derivation of sensible lifespan bounds for biological systems [4]-[5] that are reviewed in Section IV. The general ULTE equation is derived next.

First the mathematical-units based TSE entropy of (18) and the TPE ectropy of (30), as well as the physical-units based TRE entropy of (24) and the TME ectropy of (37) of LTT can be related to yield the two LTT equations:

$$\hat{H} = \hat{K}^2 \quad (40)$$

$$\hat{N} = \frac{3V}{r} = \frac{3\tau}{r\Pi} = 4\pi \left( \frac{GM}{v^2} \right)^2 = 4\pi r^2 = \frac{4v^2 \hat{A}^2}{\pi} \quad (41)$$

where all the variables in (40)-(41) were earlier defined for (18), (24), (30) and (37). In particular, for (41) we highlight the retention pace  $\Pi = \tau V$  of lifebits where  $\tau$  denotes the lifespan of a medium’s lifebits, defined as thermo-bits of interest. For example, if a medium contains a set of thermo-images and only one is of interest to us, we then say that the thermo-bits making up this thermo-image are the medium lifebits.

Expression (41) can be expressed in quantum of operation (QoO) form according to:

$$\Delta \hat{N} = 3 \frac{\Delta V}{r} = 3 \frac{\Delta \tau}{r\Pi} = 4\pi \left( \frac{GM}{v^2} \right)^2 = 4\pi \Delta r^2 = \frac{4v^2}{\pi} \Delta \hat{A}^2 \quad (42)$$

where: a)  $\Delta Y$  denotes a region in the domain of the physical variable  $Y$  that is being studied, for instance,  $\Delta \tau = 1$  day denotes the QoO for a total lifespan of  $\tau = 100$  years; and b) the radius  $r$ , pace  $\Pi$  and centripetal speed  $v$  are all assumed to remain fixed during the QoO lifespan  $\Delta \tau$ .

Moreover, obtaining the ratio of (41) over (42) the following QoO ratio of physical quantities results:

$$\frac{\hat{N}}{\Delta \hat{N}} = \frac{V}{\Delta V} = \frac{\tau}{\Delta \tau} = \left( \frac{M}{\Delta M} \right)^2 = \left( \frac{r}{\Delta r} \right)^2 = \left( \frac{\hat{A}}{\Delta \hat{A}} \right)^2 \quad (43)$$

Combining (40) and (43) the ULTE follows according to:

$$\hat{H} = \frac{\hat{S}}{k \ln 2} = f_{Med} \left( \frac{\hat{N}}{\Delta \hat{N}} = \frac{V}{\Delta V} = \frac{\tau}{\Delta \tau} = \left( \frac{M}{\Delta M} \right)^2 = \left( \frac{r}{\Delta r} \right)^2 = \left( \frac{\hat{A}}{\Delta \hat{A}} \right)^2 \right) = \hat{K}^2 \quad (44)$$

where  $f_{Med}$  is a medium dependent function that relates the mathematical-units based entropy of sources and ectropy of processors of (40) to the physical-units based entropy of retainers and ectropy of movers of (41). For instance, for either a black-hole or a photon gas  $f_{Med}$  is linear, i.e.,

$$\hat{H} = \frac{\hat{S}}{k \ln 2} = \frac{\hat{N}}{\Delta \hat{N}} = \frac{V}{\Delta V} = \frac{\tau}{\Delta \tau} = \left( \frac{M}{\Delta M} \right)^2 = \left( \frac{r}{\Delta r} \right)^2 = \left( \frac{\hat{A}}{\Delta \hat{A}} \right)^2 = \hat{K}^2 \quad (45)$$

while for the flexible phase case it would have an exponential form as will be seen in the next section.

## C.3. Linger Thermo Engines

In LTT a cyclic engine may be described using anyone of four formulations depending on the preference/objective of the engine user or designer. Two of the four formulations conceive an engine as a LTT thermodynamics engine where energy transformations occur, while the remaining two formulations conceive an engine as a LTT lingerdynamics engine where viscosity transformations occur (viscosity in viscosity Pa.sec units is the retention dual of energy in N.m units in motion). These four alternative engine descriptions borrow directly from the UC DP duality language for communication and observation systems that were used earlier to describe LIT’s CSI, SRI, SPI and CMI coders.

One of the two LTT thermodynamics engine formulations describes a channel and source integrated or CSI heat-engine that transforms a reservoir’s QoO sourced-heat  $\delta Q$  to work  $W$ , with part of  $\delta Q$  waist since lost to a sink.



The second formulation describes a sensor and retainer integrated or SRI work-engine that transforms a reservoir's QoO retained-work  $\delta W$  to heat  $Q$ , with part of  $\delta W$  waist since lost to a sink.

On the other hand, one of the two LTT linderdynamics engine formulations describes a sensor and processor integrated or SPI hover-engine that transforms a reservoir's QoO processing-hover  $\delta A$  to effort  $\Phi$ , with part of  $\delta A$  waist since lost to a sink. The second formulation describes a channel and mover integrated or CMI effort-engine that transforms a reservoir's QoO motioning-effort  $\delta \Phi$  to hover  $A$ , with part of  $\delta A$  waist since lost to a sink.

These four LTT engine formulations are now explained from a global perspective while using the four-stroke engine displayed in Fig. 3 where a fuel/air mixture is 'channeled' through a cylinder and is 'sensed' by a piston. This engine yields energy and viscosity transformations through intake, compression, power and exhaust strokes that are cyclically enabled by valves, a crankshaft and a spark plug. The LTT thermodynamics engine formulations are discussed first in C.3.1 and C.3.2, then the linderdynamics viscidities in C.3.3, and finally the LTT linderdynamics engine formulations in C.3.4 and C.3.5.

The two types of LTT thermodynamics engine formulations are now discussed.

**C.3.1. The CSI Heat-Engine Formulation and its Thermo-Source Channel-Capacity Metric:** The CSI heat-engine formulation uses our unifying UCDP duality terms to describe an engine [1]. Its basic operation is illustrated with the four-stroke engine of Fig. 3 where the engine is said to be powered by 'sourced-heat' in the fuel/air mixture that is 'communicated to the engine channel', i.e. its cylinder. The sourced-heat  $\delta Q$  is a QoO contribution defined according to:

$$\delta Q = T \Delta \hat{S} = \ln 2kT \Delta \hat{H} = T_k \Delta \hat{H} \quad (46)$$

$$T_k = \delta Q / \Delta \hat{H} = \ln 2kT \quad (47)$$

where: a)  $T$  is the medium temperature; b)  $\Delta \hat{S}$  is the QoO version of the medium's EBE of (1); c)  $\Delta \hat{H}$  is the QoO TSE; and d)  $T_k$  is the LTT-temperature in heat-energy per thermo-bit units.

The communicated sourced-heat energy is enabled by intake/exhaust valves, a crankshaft, and a spark-plug (VCS). In turn, the engine sensor, i.e., its piston, which is in harmony with crankshaft-driven fuel/air mixture motions, senses the work-state of the mixture as the sourced-heat is transformed to work  $W$  thru the crankshaft enabled strokes of the engine cycle. During the last exhaust-stroke, however, a portion of the sourced-heat energy in the fuel/air mixture cannot be transformed into useful work, since via the 'engine channel' it is 'communicated as sourced-heat waist'.

In LTT, as in classical thermodynamics, to guide the design of a CSI Heat-Engine one defines a metric. This metric is called the *thermo-source channel-capacity* with symbol  $\hat{C}$  which is a measure of the engine efficiency in

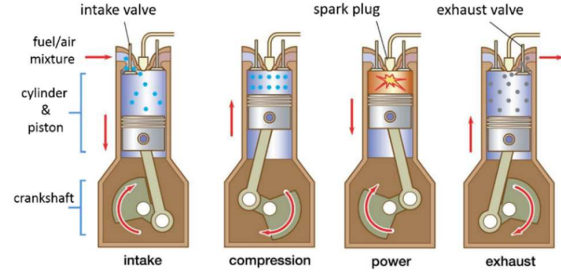


Fig. 3. The cyclic four-stroke engine

transforming the incoming sourced-heat energy  $\delta Q_{In}$  to work energy  $W$ . Its definition is:

$$\hat{C} = \frac{W}{\delta Q_{In}} = \frac{\delta Q_{In} - \delta Q_{Out}}{\delta Q_{In}} = \frac{\ln 2kT_{In} \Delta \hat{H} - \ln 2kT_{Out} \Delta \hat{H}}{\ln 2kT_{In} \Delta \hat{H}} = 1 - \frac{T_{k,Out}}{T_{k,In}} < 1 \quad (48)$$

where: 1)  $\delta Q_{In}$  and  $\delta Q_{Out}$  are the incoming and outgoing sourced-heats of the engine; 2)  $W = \delta Q_{In} - \delta Q_{Out}$  is the amount of generated work; and 3)  $\Delta \hat{H}$  is assumed to be the same for both  $\delta Q_{In}$  and  $\delta Q_{Out}$  [1].

From (48) one concludes that the CSI Heat-Engine efficiency only depends on the LTT-temperature  $T_k$  and not on  $\Delta \hat{H}$ , with the ratio  $T_{k,Out} / T_{k,In}$  being as small as possible for best engine efficiency.

**C.3.2. The SRI Work-Engine Formulation and its Thermo-Retainer Sensor-Scope Metric:** The SRI work-engine formulation uses our unifying UCDP duality terms to describe an engine. Its basic operation is once again illustrated with the four-stroke engine of Fig. 3 where the engine is now said to be powered by 'retained-work' in the fuel/air mixture that is 'observed across the engine sensor', i.e., its piston. The retained-work (or incoming mechanical work)  $\delta W$  is a QoO contribution defined according to:

$$\delta W = P \Delta V = \frac{Pr \Delta \hat{N}}{3} = P_r \Delta \hat{N} \quad (49)$$

$$P_r = \delta W / \Delta \hat{N} = Pr/3 \quad (50)$$

where: a)  $P$  is the medium pressure and  $\Delta V$  its volume increase; b)  $\Delta \hat{N}$  is the QoO version of the medium's TRE  $\hat{N}$ ; c)  $r$  is the radius of the TRE sphere; and d)  $P_r$  is the LTT-radial-pressure in work-energy per thermo- $m^2$  units.

The observed retained-work energy is enabled by the VCS. In turn, the engine channel, i.e., its cylinder, which is in harmony with valve-driven fuel/air mixture retentions, channels the heat-state of the mixture as the retained-work is transformed into heat thru the spark plug enabled strokes of the engine cycles. During the last exhaust-stroke, however, a portion of the retained-work energy in the fuel/air mixture cannot be transformed into useful heat, since via the 'engine sensor' it is 'observed as retained-work waist'.

In LTT to guide the design of a SRI Work-Engine one defines a metric. This metric is called the *thermo-retainer sensor-scope* with symbol  $\hat{j}$  which is a measure of

the engine efficiency in transforming the incoming retained-work energy to heat energy  $Q$ . Its cyclic definition is:

$$\hat{i} = \frac{Q}{\delta W_{In}} = \frac{\delta W_{In} - \delta W_{Out}}{\delta W_{In}} = \frac{\hat{P}_{In} r_{In} \Delta \hat{N} / 3 - \hat{P}_{Out} r_{Out} \Delta \hat{N} / 3}{\hat{P}_{In} r_{In} \Delta \hat{N} / 3} = 1 - \frac{P_{r,Out}}{P_{r,In}} < 1 \quad (51)$$

where: 1)  $\delta W_{In}$  and  $\delta W_{Out}$  are the incoming and outgoing retained-works of the engine; 2)  $Q = \delta W_{In} - \delta W_{Out}$  is the generated heat; 3)  $\Delta \hat{N}$  is assumed to be the same for both  $\delta W_{In}$  and  $\delta W_{Out}$ .

From (51) one concludes that the SRI Work-Engine efficiency only depends on the LTT-radial-pressure  $P_r$  and not on  $\Delta \hat{N}$ , with the ratio  $P_{r,Out} / P_{r,In}$  being as small as possible for best engine efficiency.

Since they describe the same engine from different perspectives the CSI heat-engine and SRI work-engine formulations are said to be equivalent. In either case the engine cycle is characterized by the pace  $\Pi_C$ :

$$\Pi_C = \tau_C / V_C \quad (52)$$

which denotes the duration of the engine cycle  $\tau_C$  per amount of fuel volume used  $V_C$ .

**C.3.3. The LTT Viscidities:** Viscidity with symbol  $\varpi$  with viscosity  $Pa.sec$  units is used in MRT retention problems. It is the retention dual of energy with symbol  $E$  in  $N.m$  units used in MRT motion problems. While the mass-energy equation  $E=Mc^2$  of a medium relates its mass  $M$  and energy  $E$  where  $c$  is the speed of light, the mater-viscidity equation  $\varpi = O\chi^2$  relates its mater  $O$ , in energy per pace units as noted in (12), and viscidity  $\varpi$  where  $\chi$  is the pace of dark (6). In LTT lingerdynamics there are two fundamental types of viscidity. One is the viscidity of heat energy which has been named 'hover' with symbol  $A$  and the other is the viscidity of work energy which has been named 'effort' with symbol  $\Psi$ .

In LTT lingerdynamics one studies viscidity transformations from processing-hover to effort, and from motioning-effort to hover.

In particular, 'hover' conveys the viscidity of the engine's medium when it is heated, e.g., when the fuel/air mixture is heated in the cylinder of the engine of Fig. 3. The QoO hover  $\delta A$  and the QoO heat  $\delta Q$  are thus related according to:

$$\delta A = \Pi_C \delta Q \quad (53)$$

where  $\Pi_C$  is the engine cycle pace (52).

On the other hand, 'effort' conveys the viscidity of the engine's medium when it is moved, e.g., when the fuel/air mixture is moved by the piston of the engine of Fig. 3. The QoO effort  $\delta \Psi$  and the QoO work  $\delta W$  are thus related according to:

$$\delta \Psi = \Pi_C \delta W \quad (54)$$

where  $\Pi_C$  is the engine cycle pace (52).

The two types of LTT lingerdynamics engine formulations are now discussed.

**C.3.4. The SPI Hover-Engine Formulation and its Thermo-Processor Sensor-Consciousness Metric:** The SPI hover-engine formulation uses our unifying UC DP duality terms to describe an engine. Its basic operation is once again illustrated with the four-stroke engine of Fig. 3 where the engine is now said to be powered by 'processing-hover' in the fuel/air mixture that is 'observed across the engine sensor', i.e., its piston. The processing-hover  $\delta A$  is a QoO contribution defined according to:

$$\delta A = \Pi_C \delta Q = \Pi_C T_k \Delta \hat{H} = 2 \Pi_C T_k \hat{K} \Delta \hat{K} = L \Delta \hat{K} \quad (55)$$

$$L = \delta A / \Delta \hat{K} = 2 \Pi_C T_k \hat{K} \quad (56)$$

where: a)  $\Pi_C$  is the engine cycle pace (52); b)  $T_k$  is the LTT-temperature; c)  $\Delta \hat{H}$  is the QoO TSE; d)  $\hat{K}$  is the TPE; e)  $L$  is the LTT-lingerature in hover-viscosity per lingerbor units; and f) equation (55) was derived making use of the assumption  $\Delta \hat{H} / \Delta \hat{K} \approx 2 \hat{K}$  where  $2 \hat{K}$  is the derivative  $d\hat{H} / d\hat{K}$  found from (40).

The observed hover viscosity is enabled by the VCS. In turn, the engine channel, i.e., its cylinder, which is in harmony with valve-driven fuel/air mixture retentions, channels the effort-state of the mixture as the processing-hover is transformed into effort thru the strokes of the engine cycles. During the last exhaust-stroke, however, a portion of the processing-hover viscidity in the fuel/air mixture cannot be transformed into useful effort, since via the 'engine sensor' it is 'observed as processing-hover waist'.

In LTT to guide the design of a SPI Hover-Engine one defines a metric. The metric is called the thermo-processor sensor-consciousness with symbol  $\hat{F}$  which is a measure of the engine efficiency in transforming the incoming processing-hover viscidity to effort viscidity  $\Psi$ . Its cyclic definition is:

$$\hat{F} = \frac{\Psi}{\delta A_{In}} = \frac{\delta A_{In} - \delta A_{Out}}{\delta A_{In}} = \frac{2 \Pi_C T_{k,In} \hat{K}_{In} \Delta \hat{K} - 2 \Pi_C T_{k,Out} \hat{K}_{Out} \Delta \hat{K}}{2 \Pi_C T_{k,In} \hat{K}_{In} \Delta \hat{K}} = 1 - \frac{L_{Out}}{L_{In}} < 1 \quad (57)$$

where: 1)  $\delta A_{In}$  and  $\delta A_{Out}$  are the incoming and outgoing processing-hovers of the engine; 2)  $\Psi = \delta A_{In} - \delta A_{Out}$  is the generated effort; and 3)  $\Delta \hat{K}$  is assumed to be the same for both  $\delta A_{In}$  and  $\delta A_{Out}$ .

From (57) one concludes that the SPI Hover-Engine efficiency only depends on the LTT-lingerature  $L$  and not on  $\Delta \hat{K}$ , with the ratio  $L_{Out} / L_{In}$  being as small as possible for best engine efficiency.

**C.3.5. The CMI Effort-Engine Formulation and its Thermo-Mover Channel-Stay Metric:** The CMI effort-engine uses our unifying UC DP duality terms to describe an engine. Its basic operation is once again illustrated with the four-stroke engine of Fig. 3 where the engine is now said to be powered by 'motioning-effort' in the fuel/air mixture that is 'communicated to the engine channel', i.e., its cylinder.

The moving-effort  $\delta \Psi$  is a QoO contribution defined according to:

$$\delta\mathcal{P} = \Pi_C \delta W = \Pi_C P_r \Delta \hat{N} = 8G\Pi_C M P_r \hat{\Delta} \Delta \hat{A} / \pi = \hat{P} \Delta \hat{A} \quad (58)$$

$$\hat{P} = \delta\mathcal{P} / \Delta \hat{A} = 8G\Pi_C M P_r \hat{A} / \pi \quad (59)$$

where: a)  $\Pi_C$  is the engine cycle pace (52); b)  $P_r$  is the LTT-radial-pressure; c)  $\Delta \hat{N}$  is the QoO TRE; d)  $\hat{A}$  is the TME; e)  $P$  is medium pressure; f)  $M$  is medium mass; e)  $\hat{P}$  is the LTT-pressure in effort-viscosity per linger-sec units; and f) equation (58) was derived under the assumption  $\Delta \hat{N} / \Delta \hat{A} \approx 8GM\hat{A} / \pi r$  with  $8GM\hat{A} / \pi r$  being the derivative  $d\hat{N} / d\hat{A}$  found via (36) and (41).

The communicated effort viscosity is enabled by the VCS. In turn, the engine sensor, i.e., its piston, which is in harmony with crankshaft-driven fuel/air mixture motions, senses the processing-state of the mixture as the motioning-effort is transformed into hover thru the spark plug enabled strokes of the engine cycles. During the last exhaust-stroke, however, a portion of the motioning-effort viscosity in the fuel/air mixture cannot be transformed into useful hover, since via the 'engine channel' it is 'communicated as motioning-effort waist'.

In LTT to guide the design of a CMI Effort-Engine one defines a metric. The metric is called thermo-mover channel-stay with symbol  $\hat{T}$  which is a measure of the engine efficiency in transforming the incoming motioning-effort viscosity to hover viscosity  $\mathcal{A}$ . Its cyclic definition is:

$$\hat{T} = \frac{\mathcal{A}}{\delta\mathcal{P}_{in}} = \frac{\delta\mathcal{P}_{in} - \delta\mathcal{P}_{out}}{\delta\mathcal{P}_{in}} = \frac{\frac{8}{\pi} G\Pi_C M_{in} P_{in} A_{in} \Delta \hat{A} - \frac{8}{\pi} G\Pi_C M_{out} P_{out} A_{out} \Delta \hat{A}}{\frac{8}{\pi} G\Pi_C M_{in} P_{in} A_{in} \Delta \hat{A}} = 1 - \frac{\hat{P}_{out}}{\hat{P}_{in}} < 1 \quad (60)$$

where: 1)  $\delta\mathcal{P}_{in}$  and  $\delta\mathcal{P}_{out}$  are the incoming and outgoing motioning-efforts of the engine; 2)  $\mathcal{A} = \delta\mathcal{P}_{in} - \delta\mathcal{P}_{out}$  is the generated hover; 3)  $\Delta \hat{A}$  is assumed to be the same for both  $\delta\mathcal{P}_{in}$  and  $\delta\mathcal{P}_{out}$ .

From (60) one concludes that the CMI Effort-Engine efficiency only depends on the LTT-pressure  $\hat{P}$  and not on  $\Delta \hat{A}$ , with the ratio  $\hat{P}_{out} / \hat{P}_{in}$  being as small as possible for best engine efficiency.

Since they describe the same engine from different perspectives the SPI hover-engine and CMI effort-engine formulations are said to be equivalent. In either case the engine cycle is characterized by the cycle pace  $\Pi_C$  of (52).

### III. THE UNIVERSAL DOF-TSE EQUATION

The UDTE of (2) is now shown to agree with LTT-based entropy expressions for an uncharged, nonrotating black-hole, a photon gas, and a flexible phase homogenous-medium.

#### A. The Black-hole Medium Case

The uncharged, nonrotating black-hole medium is spherical in shape and thus inherently satisfies the TRE spherical efficiency requirement. The TSE for this case is the LTT-Bekenstein-Hawking black-hole entropy [4]:

$$\hat{H}_{BH} = \frac{\hat{S}_{BH}}{k \ln 2} = \frac{c\chi \hat{N}_{BH}}{1920 \ln 2} = \frac{c^3 (4\pi r_{BH}^2)}{4G\hbar \ln 2} = \frac{4\pi GM_{BH}^2 c^4}{\hbar c^5 \ln 2} = \frac{4\pi GE_{BH}^2}{\hbar c^5 \ln 2} \quad (61)$$

$$\hat{N}_{BH} = 4\pi r_{BH}^2 \quad (62)$$

$$r_{BH} = 2GM_{BH} / c^2 = 2GE_{BH} / c^4 \quad (63)$$

where: a)  $\chi$  is the pace of dark in a black-hole (6) ; b)  $\hat{N}_{BH}$  is the black-hole TRE; and b) expression (63) is the Schwarzschild radius of the event-horizon of a black-hole [2].

Next it is shown that the black-hole TSE equation (61) satisfies the UDTE of (2). First the black-hole temperature  $T_{BH}$  is expressed in terms of the total black-hole energy  $E_{BH}$  making use of (61) to yield:

$$T_{BH} = \left( \partial \hat{S}_{BH} / \partial E_{BH} \right)^{-1} = \hbar c^5 / 8k\pi GE_{BH} \quad (64)$$

Then using (64) in (61) one finds the sought after UDTE relationship shown on the right hand side of the equation below

$$\hat{H}_{BH} = \frac{\hat{S}_{BH}}{k \ln 2} = \frac{c\chi (4\pi r_{BH}^2)}{1920 \ln 2} = \frac{M_{BH} c^2}{2 \times \ln 2 k T_{BH}} \quad (65)$$

where as noted in the introductory section the first '2' appearing in  $2 \times \ln 2 k T_{BH}$  corresponds to the two degrees of freedom available for particle motion on the surface (or event horizon) of the black-hole.

#### B. The Photon Gas Medium Case

The photon gas medium is not necessarily spherical in shape. As a result, the TRE spherical efficiency requirement of LTT must be satisfied by expressing its actual volume value as if resulting from a spherical medium. The TSE for this photon case is [2]-[4]:

$$\hat{H}_{PG} = \frac{S_{PG}}{k \ln 2} = \frac{4\pi^2 (kT_{PG})^3}{45c^3 \hbar^3 \ln 2} V_{PG} = \frac{16\pi^3 (kT_{PG})^3}{135c^3 \hbar^3 \ln 2} V_{PG} = \frac{16\pi^3 (kT_{PG})^3 G^3 E_{PG}^2}{135c^3 \hbar^3 \ln 2 c^6 v_{PG}^6} \quad (66)$$

$$V_{PG} = 4\pi r_{PG}^3 / 3 = r_{PG} \hat{N}_{PG} / 3 \quad (67)$$

$$r_{PG} = GM_{PG} / v_{PG}^2 = GE_{PG} / v_{PG}^2 c^2 \quad (68)$$

where: a) equation (67) expresses the unknown volume shape of the photon gas as spherical with its surface area given by the TRE  $\hat{N}_{PG}$ ; and b) equation (68) expresses the radius of the TRE sphere in terms of the centripetal speed of a point-mass at the radial distance  $r_{PG}$  from the medium's point-mass  $M_{PG}$  of mass-energy  $E_{PG} = M_{PG} c^2$ .

Next it is shown that the photon-gas TSE equation (66) satisfies the UDTE of (2). First the photon-gas temperature  $T_{PG}$  is expressed in terms of the total photon-gas energy  $E_{PG}$  making use of (66) to yield:

$$T_{PG} = \left( \partial S_{PG} / \partial E_{PG} \right)^{-1} = 135c^9 \hbar^3 v_{PG}^6 / 48k\pi^3 (kT_{PG})^3 G^3 E_{PG}^2 \quad (69)$$

Then using (69) in (66) one finds the sought after UDTE relationship shown on the right hand of the equation below

$$\hat{H}_{PG} = \frac{S_{PG}}{k \ln 2} = \frac{4\pi^2 (kT_{PG})^3}{45c^3 \hbar^3 \ln 2} V_{PG} = \frac{16\pi^3 (kT_{PG})^3 G^3 E_{PG}^2}{135c^3 \hbar^3 \ln 2 c^6 v_{PG}^6} = \frac{M_{PG} c^2}{3 \times \ln 2 k T_{PG}} \quad (70)$$

where as noted in the introductory section the '3' in  $3 \times \ln 2 k T_{PG}$  denotes the  $N_{DoF}$  of photons moving in 3D dimensional space.

#### C. The Flexible Phase Medium Case

The flexible phase medium is not necessarily spherical in shape. As a result the TRE spherical efficiency requirement of LTT must be satisfied by expressing its actual volume value as if resulting from a spherical medium. The TSE for this case is the LTT flexible phase entropy first formulated in [5] according to:

$$\hat{H}(c_h(\eta)) = \frac{\hat{S}(c_h(\eta))}{k \ln 2} = \frac{J(c_h(\eta))}{\ln 2} \ln \left( \frac{e^{c_h(\eta) \eta} q(c_h(\eta)) \left( \frac{E}{c_h(\eta) k T J(c_h(\eta))} \right)^{c_h(\eta)}}{J^{\eta}(c_h(\eta))} \right) \quad (71)$$

$$V = 4\pi r^3 / 3 = r \hat{N} / 3 \quad (72)$$

$$r = GM / v^2 = GE / v^2 c^2 \quad (73)$$

$$c_h(\eta) = N_{DoF}(\eta) / 2 \quad (74)$$

$$J(c_h(\eta)) = E / c_h(\eta)kT = Mc^2 / c_h(\eta)kT \quad (75)$$

$$\eta = \eta_{Max} \frac{c_h(\eta) - 3/2}{c_{h,Max} - 3/2} = \eta_{Max} \frac{N_{DoF}(\eta) - 3/2}{N_{DoF,Max} - 3/2} \quad (76)$$

$$q(c_h(\eta)) = q^e q^t (q^r q^v)^{\frac{c_h(\eta) - 3/2}{c_{h,Max} - 3/2}} \quad (77)$$

$$q^e = \sum_{\text{Electronic Energies } \epsilon_i^e} g_i e^{-\epsilon_i^e / kT} \approx g_0 e^{-\epsilon_0^e / kT} = g_0 \quad (78)$$

$$q^t = \sum_{\text{Translational Energies } \epsilon_i^t} g_i e^{-\epsilon_i^t / kT} \approx V \left( \frac{mkT}{2\pi\hbar^2} \right)^{3/2} \quad (79)$$

$$q^r = \sum_{\text{Rotational Energies } \epsilon_i^r} g_i e^{-\epsilon_i^r / kT} \approx \frac{2IkT}{\sigma\hbar^2} \quad (80)$$

$$q^v = \sum_{\text{Vibrational Energies } \epsilon_i^v} g_i e^{-\epsilon_i^v / kT} = \frac{1}{1 - e^{-\hbar\nu / kT}} \begin{cases} 1, & kT \ll \hbar\nu \\ kT / \hbar\nu, & kT \gg \hbar\nu \end{cases} \quad (81)$$

where: a) equation (72) conveys the unknown shape of the volume of a flexible phase medium as spherical with its surface area given by the TRE  $\hat{N}$ ; b) equation (73) expresses the radius of the TRE sphere in terms of the centripetal speed of a point-mass at the radial distance  $r$  from the medium's point-mass  $M$  with mass-energy  $E=Mc^2$ ; c) equation (74) defines the heat capacity  $c_h(\eta)$  of the medium as the ratio of the medium's degrees of freedom  $N_{DoF}(\eta)$  over two; d) equation (75) defines the number of *thermotes*  $J(c_h(\eta))$  for the medium where the mass-energy of a thermote is temperature dependent and given by:

$$e_T = m_T c^2 = c_h(\eta)kT; \quad (82)$$

e) equation (76) defines a DoF-coupling-constant  $\eta$  reflecting a decrease in the DoF heat-capacity of a homogeneous medium, such as water, when chemical reactions give rise to new medium particles or molecules such as hydrogen-ions or protons, electrons, and free radicals that further interact among themselves; f)  $N_{DoF,Max}$  in (76) denotes the maximum amount of degrees of freedom for medium particles which occurs when the medium is homogeneous, e.g., it will be assumed here to be '5' for water at 310 K; g) equation (76) defines  $\eta$  as the product of its maximum value  $\eta_{Max}$ , found by setting  $N_{DoF}(\eta)$  equal to  $N_{DoF,Max}$  in (76), and the less than or equal to one DoF ratio given by  $(N_{DoF}(\eta)-3)$  over  $(N_{DoF,Max}-3)$  with '3' denoting the minimum DoF case for particle 3D translations only; h)  $q(c_h(\eta))$  of (77) is the partition function of a flexible phase medium defined as the product of electronic  $q^e$ , translational  $q^t$ , rotational  $(q^r)^{(c_h(\eta)-3/2)/(c_{h,Max}-3/2)}$  and vibrational  $(q^v)^{(c_h(\eta)-3/2)/(c_{h,Max}-3/2)}$  factors that depend on the heat-capacity of the medium; i) in (77) thru (81)  $q^e$ ,  $q^t$ ,  $q^r$ , and  $q^v$  are defined for diatomic molecules such as the H<sub>2</sub>O molecules in water at 310 K that will be used in this paper to model biological mediums; j)  $m$  in (79) is the molecular mass, e.g.,  $3 \times 10^{-26}$  kg for H<sub>2</sub>O; k)  $g_0$  in (78) is the degeneracy of the ground zero energy state, i.e.,  $\epsilon_0^e = 0$ , this degeneracy will be assumed to be one for water at 310 K; l) ' $I$ ' is the average moment of inertia of the particle, e.g.,  $2 \times 10^{-47}$  kg.m<sup>2</sup>; m)  $\sigma$  is the symmetry number of the molecule, e.g., 2 for H<sub>2</sub>O; o)  $\nu$  is the vibration frequency of the particle, e.g., the condition  $kT \ll \hbar\nu$  will be assumed here for water at 310 K which results in the unity vibrational factor of  $q^v=1$  noted in (81); and p) the number of thermotes expression (75) follows from making use of (71) to find the flexible phase  $T$  in terms of  $E=Mc^2$  according to:

$$T = \left( \partial \hat{S}(c_h(\eta)) / \partial E \right)^{-1} = Mc^2 / c_h(\eta)kJ(c_h(\eta)) \quad (83)$$

**C.1. Application of the UDTE to the flexible phase TSE:** The UDTE of (2) is for homogeneous mediums only. As a result, the TSE of (71) only satisfies (2) for the maximum DoF case. Thus

when (2) and (75) are applied to (71) for the maximum DoF case one finds:

$$\hat{H} = \frac{\hat{S}}{\ln 2k} = \frac{Mc^2}{\ln 2c_h kT} \ln \frac{e^{c_h \eta_{Max}} q}{(Mc^2 / \ln 2c_h kT)^{\eta_{Max}}} = \frac{Mc^2}{2 \ln 2c_h kT} \quad (84)$$

where  $\hat{H} = \hat{H}(c_{h,Max})$ ,  $\hat{S} = \hat{S}(c_{h,Max})$ ,  $c_h = c_{h,Max} = N_{DoF,Max}/2$  and  $q = q(c_{h,Max})$ . In particular, the  $\eta_{Max}$  in (84) is the maximum value of the DoF-coupling-constant  $\eta$  when  $c_h(\eta)$  becomes  $c_{h,Max}$  in (76). One can now use (84) to find an expression for  $\eta_{Max}$ . To find  $\eta_{Max}$  one first notes from (84) that the following condition must apply:

$$\ln \frac{e^{c_{h,Max}} q(c_{h,Max})}{(Mc^2 / e c_{h,Max} kT)^{\eta_{Max}}} = \frac{1}{2} \quad (85)$$

Expression (85) can now be solved for  $\eta_{Max}$  to yield:

$$\eta_{Max} = \left( \frac{1}{2} - c_{h,Max} - \ln q(c_{h,Max}) \right) / \ln \left( \frac{e c_{h,Max} kT}{Mc^2} \right) \quad (86)$$

In the next section the flexible phase TSE of (71) together with the newly established  $\eta_{Max}$  expression of (86) is applied to the derivation of three alternative LTT driven theoretical bounds for biological lifespan. It is perhaps of interest to note that in [22]-[23] the *first use of a water medium to study the lifespan of a biological medium* was performed. However, this study was only of a preliminary nature since the entropy used in [22]-[23] for the water medium was not the LTT based TSE of (71), first offered in [5], but rather the classical entropy expression for an ideal gas [36].

#### IV. THEORETICAL LIFESPAN STUDY

In this section novel LTT statistical bounds are derived from a theoretical study of the lifespan of thermo-bits of interest or lifebits in a flexible phase water medium whose basic homogeneous state can be altered by chemical reactions. This kind of medium will be used to model a biological medium such as our own since 98.73% of our molecules are of water, while only: 0.74% are of other inorganic molecules; 0.475% for lipids; 0.044% for other organic molecules; 0.011% for proteins;  $3 \times 10^{-5}$  % for RNA; and  $3 \times 10^{-11}$  % for DNA [6], [23]. In addition, the LTT lifebits of our theoretical lifespan equations will be assumed to model those of the mitochondria DNA ones (plus possibly other molecule types) according to *Harman's free radical theory of aging*, where these lifebits should be gradually lost due to free radical driven mutations [7]. More specifically, the theoretical lifespan bounds to be discussed advance a performance limit for the lifespan of the lifebits of our biological medium whose mass-energy  $E=Mc^2$  is regulated or kept constant throughout. Just as significantly, although the weight contributed by water molecules is approximately 65% of our total mass [6], [23], it would be assumed here that the remaining 35% of  $M$  also arises from water molecules that are, however, subjected to chemical reactions giving rise to high energy electrons, hydrogen ions, free radicals, etc.. The chemical reactions among these molecules would thus be assumed to modulate the DoF heat capacity of our water-based flexible phase medium. The value of  $E=Mc^2$  would not vary in our lifespan computations since the incoming operating heat-energy for daily system consumption  $\delta Q$  would be assumed to be cancelled by the outgoing radiation heat-energy  $\diamond Q$  [3] where a part of  $\diamond Q$  would correspond to unrecoverable lifebits that sustain life.

The development begins with the application of the ULTE of (44) to a flexible phase medium. This application is then used to derive three different *'theoretical lifespan bounds'* for water-based mediums such as our own. In this study three basic

assumptions are made. Firstly, that the QoO lifespan  $\Delta\tau$  is the duration of one day. Secondly, that the QoO mass  $\Delta M$  denotes our nutritional consumption rate (NCR) or energy use. This NCR is responsible for the creation of high energy electrons during our metabolic functions. For more than half a century scientific evidence has surfaced that the aging process may be significantly driven by mitochondria-DNA mutations that are triggered by free radicals [7]-[9]. The mitochondria is the power house of the cell where ATP molecules are synthesized to provide the necessary energy for performing basic biological functions. During this synthesis the continuous generation of Hydrogen ions  $H^+$ , i.e., protons, by the mitochondria through high energy electron production is essential. In our theoretical study it will be shown that sensible theoretical adult lifespan bounds that naturally surface from our ULTE support this aging theory. Thirdly and last  $\tau$  denotes the lifespan of thermo-bits of interest or *lifebits*. In our discussions it will be assumed that in biological systems these lifebits would be found in the mitochondria-DNA, and are lost when irreversible ‘aging causing’ mitochondria-DNA mutations occur. These irreversible aging causing mutations are modeled by us as lifebits that are lost daily until the cells lose their ability to produce enough ATP molecules to sustain life. The limit in lifespan suggested by Harman’s theory [7] and also LTT, as will be seen, is further backed by statistical observations on human lifespan just reported in [38]-[39]. Here  $\tau$  is called *theoretical adult lifespan* with the term ‘adult’ indicating that the lifespan investigated is that of an ‘adult individual’ whose mass  $M$  is assumed to be time invariant, which in turn implies that to the value of  $\tau$  a childhood lifespan must be added, say of 18 years, to derive a total lifespan.

The application of the flexible phase (FP) TSE of (71) to the ULTE of (44) leads to the following flexible phase (FP) ULTE:

$$\begin{aligned} \ddot{H} &= \frac{\ddot{S}}{k \ln 2} \\ &= \ddot{J} \log_2 \left( \frac{e^{c_h(\eta)^{\eta}} q(c_h(\eta))}{\ddot{J}^{\eta}} = \frac{\dot{N}}{\Delta \dot{N}} = \frac{V}{\Delta V} = \frac{\tau}{\Delta \tau} = \left( \frac{M}{\Delta M} \right)^2 = \left( \frac{r}{\Delta r} \right)^2 = \left( \frac{\dot{A}}{\Delta \dot{A}} \right)^2 \right) = \ddot{K}^2 \end{aligned} \quad (87)$$

where  $\ddot{H} = \hat{H}(c_h(\eta))$ ,  $\ddot{S} = \hat{S}(c_h(\eta))$ ,  $\ddot{J} = J(c_h(\eta))$ ,  $\ddot{K} = \hat{K}(c_h(\eta))$  and the maximum value for  $\eta$ , i.e.,  $\eta_{Max}$ , required to obtain  $\eta$  from (76) is found via the UDTE based  $\eta_{Max}$  equation (86).

The FP-ULTE of (87) can now be used to find its QoO version. The QoO FP (or QFP) ULTE would model the heat-energy  $\delta Q$  that is added to the medium for its daily consumption (a similar amount of energy  $\diamond Q$  will be assumed to leave the medium as thermal radiation [3]). Firstly, the thermodynamics QoO heat-energy expression  $\delta Q = T \Delta \dot{S}$  [1] where  $T$  is temperature, is used to derive an initial QoO TSE thermal equation given by:

$$\Delta \dot{H} = \Delta \dot{S} / \ln 2k = \delta Q / \ln 2kT. \quad (88)$$

Next  $\ddot{H} = \ddot{J} \log_2 (e^{c_h(\eta)^{\eta}} q(c_h(\eta)) / \ddot{J}^{\eta} = V / \Delta V)$  from (87) is used to obtain a second expression for  $\Delta \dot{H}$  in terms of  $\ddot{H}$  as follows:

$$\begin{aligned} \Delta \dot{H} &= \ddot{H} \left| \frac{\ddot{J} \rightarrow \ddot{J} + d\ddot{J}}{\ddot{J}^{\eta} \rightarrow \Delta \ddot{J}^{\eta} (\ddot{J} + d\ddot{J}) / \ddot{J}^{\eta}} \right. \\ &\quad \left. - \ddot{H} = (\ddot{J} + d\ddot{J}) \log_2 \left( \frac{V + dV}{\Delta V \times ((\ddot{J} + d\ddot{J}) / \ddot{J})^{\eta}} \right) - \ddot{J} \log_2 \left( \frac{V}{\Delta V} \right) \right. \\ &\quad \left. \Delta V / V = \ddot{J}^{\eta} / e^{c_h(\eta)^{\eta}} q(c_h(\eta)) \right. \end{aligned} \quad (89)$$

where: a)  $\Delta V$  denotes a QoO volume that does not change the volume of the medium  $V$ ; and b)  $dV$  is an incremental volume that increases the medium volume  $V$  from day to day. The incremental volume  $dV$  is related to the thermotes added daily  $dJ$  according to:

$$PdV = kTdJ \quad (91)$$

where  $P$  is the thermote pressure associated with the  $J$  thermotes of the flexible phase medium according to:

$$PV = kT\ddot{J} \quad (92)$$

After some algebraic manipulations expression (89) the following result is derived:

$$\Delta \dot{H} = (\ddot{J} + d\ddot{J}) \log_2 ((V + dV) / \Delta V) - \ddot{J} \log_2 (V / \Delta V) + dJ \log_2 F(\eta) \quad (93)$$

$$F(\eta) = ((\ddot{J} + d\ddot{J}) / \ddot{J})^{-\eta(\ddot{J} + d\ddot{J}) / d\ddot{J}} \quad (94)$$

Next taking the ratio of (91) over (92) to obtain  $dV / V = dJ / \ddot{J}$  and also after further algebraic manipulations of (93) one obtains:

$$\Delta \dot{H} = dJ \log_2 \left( F(\eta) \frac{V}{\Delta V} \left( 1 + \frac{dV}{V} \right)^{\frac{\ddot{J} + d\ddot{J}}{d\ddot{J}}} \right) = dJ \log_2 \left( F(\eta) \frac{V}{\Delta V} \left( 1 + \frac{dJ}{\ddot{J}} \right)^{\frac{\ddot{J} + d\ddot{J}}{d\ddot{J}}} \right) \quad (95)$$

Expression (94) together with (88) can then be used in (95) to yield:

$$\Delta \dot{H} = \Delta \dot{S} / \ln 2k = \delta Q / \ln 2kT = dJ \log_2 (B(\eta) V / \Delta V) \quad (96)$$

$$B(\eta) = ((\ddot{J} + d\ddot{J}) / \ddot{J})^{(1-\eta)(\ddot{J} + d\ddot{J}) / d\ddot{J}} \quad (97)$$

Finally using in (96) the logarithmic argument appearing in (87) that relates  $V / \Delta V$  to other physical quantities, the following QoO Flexible Phase (QFP) ULTE is arrived at:

$$\begin{aligned} \Delta \dot{H} &= \frac{\Delta \dot{S}}{\ln 2k} = \frac{\delta Q}{\ln 2kT} \\ &= dJ \left( \log_2 B(\eta) + \log_2 \left( \frac{e^{c_h(\eta)^{\eta}} q(c_h(\eta))}{\ddot{J}^{\eta}} = \frac{\dot{N}}{\Delta \dot{N}} = \frac{V}{\Delta V} = \frac{\tau}{\Delta \tau} = \left( \frac{M}{\Delta M} \right)^2 = \left( \frac{r}{\Delta r} \right)^2 = \left( \frac{\dot{A}}{\Delta \dot{A}} \right)^2 \right) \right) \end{aligned} \quad (98)$$

The QFP-ULTE can now be used to derive three different types of theoretical adult lifespan bounds which are next illustrated with a human lifespan application. In this application the QFP-ULTE will assume for our medium a flexible phase water medium at 310 K, this temperature approximates that of our bodies. Moreover, when computing the partition function  $q(c_h(\eta))$  of the diatomic  $H_2O$  water molecules at 310 K it will be assumed that  $kT \ll h\nu$  which results in the vibrational factor  $q^v$  of (81) being equal to 1. This condition, in turn, leads us to  $N_{DoF,Max} = 5$  as the maximum value that the medium’s degrees of freedom can attain and  $c_{h,Max} = 5/2$  as the maximum heat-capacity value. In particular we assume that: a) the  $H_2O$  symmetry number  $\sigma$  is two; b) the  $H_2O$  average moment of inertial ‘ $I$ ’ is  $4 \times 10^{-47} \text{ kg}\cdot\text{m}^2$ ; c) the water density  $M/V$  is  $1,000 \text{ kg}/\text{m}^3$ ; d) the  $H_2O$  mass  $m$  is  $3 \times 10^{-26} \text{ kg}$ ; e) the degeneracy of the ground state  $g_0$  of water is 1; and f) the QoO lifespan  $\Delta\tau$  is of 1 day or 1/365 in year units.

Three *sensible LTT lifespan bounds or limits* are noted next to naturally surface for ‘healthy’ adult individuals whose mediums are modeled by the QFP-ULTE of (98). These are:

1) Firstly, the ‘macro’ nutritional consumption rate or *NCR theoretical adult lifespan bound* given by:

$$\tau = \Delta\tau (M / \Delta M)^2 \quad (99)$$

where  $\tau$  is the theoretical adult lifespan,  $M$  is the mass of the adult individual, and  $\Delta M$  is his NCR according to:

$$\Delta M = \delta Q / \Theta \mu \quad (100)$$

where  $\delta Q$  is the ‘heat energy’ in *Joule* units added to the medium and  $\Theta = 5,000 \text{ kcal}/\text{kg}$  and  $\mu = 4.18 \text{ kJ}/\text{kcal}$  are conversion factors. It is assumed here that a similar amount of heat energy  $\diamond Q$  is radiated to the medium surroundings as thermal radiation [3], i.e.  $\diamond Q = \delta Q$ . This incoming/outgoing mass-energy regulation condition keeps the total mass-energy of the medium  $E = Mc^2$  constant from day to day. In particular, one can readily convert the NCR from *kg* to *kcal* units by multiplying  $\Delta M$  by  $\Theta$ . Table I shows for three cases of  $M$  and four of  $\tau$  (18 years of childhood are also added during which time the individual steadily increasing his mass until it reaches the assumed constant adult mass of  $M$ ), the corresponding QoO mass

$\Delta M$  or NCR, pace of lifespan  $lT = \tau/V$  and the anti-gravitational centripetal speed  $v = (GM/r)^{1/2}$  for particles on the medium spherical surface according to the TRE of (23). The table displays theoretical total lifespan bounds as a function of the NCR or  $\Delta M$  that are quite reasonable in their values. For instance, for the 70 kg (or 154 lbs) case with a 2,024 kcal daily diet a lifespan bound of 100 yrs is derived. This is a sensible result, since, for instance, in the USA the recommended diet for a 70 kg individual is of 2,000 kcal/day, while the lifespan of the USA population is approximately 80 years (it includes all kinds of deaths and weights), which is 20 years less than the assumed lifespan bound of 100 years (in [38]-[39] evidence for a lifespan bound around 115 is claimed). Moreover, one also notices that an improved theoretical lifespan bound, i.e., from 80 to 100 years, is achieved by a healthy 70 kg individual that has his NCR reduced from 2,327 to 2,024 kcal/day. This result is also sensible since it would imply a lesser creation per day of high energy electrons, which would in turn lead to a decreased number of free radicals and thus mitochondria-DNA mutations.

2) The second lifespan bound is the ‘micro’ degrees of freedom or DoF theoretical adult lifespan given by:

$$\tau = \Delta \tau \frac{g_0 (ekT)^{c_h(\eta)} M}{1000(2\pi\hbar^2/m)^{3/2} (\pi\hbar^2 \sigma kT / 2l)^{\frac{c_h(\eta)-3/2}{2}}} \left( \frac{c_h(\eta)kT}{Mc^2} \right)^{\eta_{max}(c_h(\eta)-3/2)} \quad (101)$$

$$\eta_{max} = \left( -2 - \ln \frac{2g_0 M l (m/2)^{3/2} (kT)^{5/2}}{1000\pi^{3/2} \hbar^5 \sigma} \right) / \ln \left( \frac{5kT}{2Mc^2} \right) \quad (102)$$

where (101) arises from using in the QFP-ULTE of (98) the LTT partition function (77)-(81) with  $q^v=1$ ,  $c_{h,Max}=5/2$  and the mass-density for water given by  $\rho=M/V=1,000 \text{ kg/m}^3$ . Similarly, the  $\eta_{max}$  expression of (102) surfaces from the use in (86) of (77)-(81) with  $q^v=1$ ,  $c_{h,Max}=5/2$  and  $M/V=1,000 \text{ kg/m}^3$ . Table II shows for three cases of  $M$  and four of  $\tau$ , where 18 years of childhood are also added, the corresponding  $\eta_{max}$  and  $c_h(\eta)$  values where  $\eta_{max}$  is noted to always be less than one. The table displays bounds for expected total lifespan as a function of DoF heat capacity values. The different cases displayed show lifespan bound changes from 60 to 120 years as the  $N_{DoF}(\eta)$  value of the flexible phase medium decreases from 4.708 when  $c_h(\eta)=2.354$  to 4.682 when  $c_h(\eta)=2.341$ . This result conveys that a small change in the number of degrees of freedom of the water-based medium can produce a significant change in the theoretical lifespan bound.

3) The third and last lifespan bound is specified by the mixed micro/macro DoF-NCR theoretical adult lifespan equations:

$$\tau = \Delta \tau e^{\frac{\partial Q}{kTdJ} - \frac{\partial Q}{PdV} - \frac{\partial \mu_{\Delta M}}{kTdJ}} / B(\eta) = \Delta \tau e^{\frac{\partial Q}{kTdJ} - \frac{\partial Q}{dW} - \frac{\partial \mu}{kT} dM} / B(\eta) \quad (103)$$

$$B(\eta) = \left( 1 + c_h(\eta)kTdJ / Mc^2 \right)^{1 - \eta_{max} \frac{c_h(\eta) - 3/2}{c_{h,Max} - 3/2}} \left( 1 + \frac{Mc^2}{c_h(\eta)kTdJ} \right) \quad (104)$$

$$dM = \Delta M / dJ \quad (105)$$

$$dW = PdV = kTJdV / V = kTdJ \quad (106)$$

$$c_h(\eta) = N_{DoF}(\eta) \times 1/2 = N_{DoF} \times dMC_h(\eta) / k \quad (107)$$

$$N_{DoF}^2 = 4.4182 \quad (108)$$

where: a) the  $\tau$  eqs. (103) and (104) were found via (75), (91)-(92), (96)-(98) and (100); b) the  $B(\eta)$  of eq. (104) is  $\eta$  dependent but with a value around 1.325 for all of our illustrative cases; c)  $dM$  of (105) is a NCR per thermote rate in kg units whose value monotonically increases as  $\tau$  increases in value; d) the  $dW$  eq. of (106) denotes expansion work  $PdV$  whose increase has a negative impact in adult lifespan according to the exponential term  $e^{\partial Q/dW}$  of (103); e) the  $c_h(\eta)$  variable in (107) denotes a ‘normalized’ specific heat-capacity whose value is 3,470 J/kg.K.DoF when an

individual’s mass is 70 kg and has a total lifespan of 80 yrs or  $\tau=62$  yrs (the average specific heat capacity of the human body is 3,470 [40]); and f)  $N_{DoF}$ , whose value is less than  $N_{DoF,Max} = 5$ , is a ‘normalizing’ DoF number that for a 70 kg individual with a total lifespan of 80 yrs yields  $C_h(\eta)=3,470 \text{ J/kg.K.DoF}$  (for this case  $\Delta M=0.4653 \text{ kg}$  or 2,327 kcal/day,  $c_h(\eta)=2.3478$  and  $dJ=2.2007 \times 10^{26}$  thermotes). As was done in Tables I and II for the macro NCR and micro DoF cases, Tables III and IV show three cases of  $M$  and four of  $\tau$  where 18 years of childhood are also added. In Table III the percentage amount of the different types of daily metabolism energy contributions to  $\partial Q$  are displayed. These contributed energies are: the internal energy  $dE=c_h(\eta)kTdJ$ ; the expansion work  $dW=PdV=kTdJ$ ; the enthalpy energy  $d\bar{H}=dE+dW$ ; and the electro-chemical Gibbs energy  $-dG=\partial Q-dH$ . Three results are highlighted in Table III. Firstly, it is noted that the % of these energies in  $\partial Q$  does not change as the individual’s mass is changed from 50 to 70 and then 100 kg. Secondly, a higher lifespan is found to be linked to a more efficient use of the electro-chemical Gibbs energy  $-dG$ . For instance, one notices that it is 69.1% of  $\partial Q$  for an adult lifespan of 102 yrs, while it is only 66.3% of  $\partial Q$  for an adult lifespan of 42 yrs. Thirdly, a higher lifespan is linked to a more efficient use of the expansion work  $dW$  which is 9.25% for a 102 yrs adult lifespan and 10.05% for a 42 yrs one.

Next in Table IV sensible results are tabulated for expected total lifespan as a function of the NCR per thermote rate  $dM$  of (105) and the variable  $B(\eta)$  of (104). The different cases displayed on Table IV are sensible in the sense that ‘adult’ lifespan changes from 42 to 102 years are linked to  $dM$  changes that are consistent with ‘mitochondria’ behavior in a real-world biological system scenario [7]-[8] as we now discuss. First it is noted that for a 42 yrs adult lifespan  $dM$  would be  $2.0308 \times 10^{-27} \text{ kg}$ , while for a 102 yrs adult lifespan it would be around 10% larger in value, i.e.,  $2.2143 \times 10^{-27} \text{ kg}$ . In addition, for all the body masses considered  $dM$  does not change in value for any given adult lifespan even though the thermote rate  $dJ$  varies considerably. For instance, for an adult lifespan of 102 years  $dJ$  is  $1.1702 \times 10^{26}$  for a 50 kg individual while it is approximately 100% more, i.e.,  $2.342 \times 10^{26}$ , for a 100 kg one. Even more enlightening is the revelation that for all the considered lifespan cases the derived  $dM$  value is always greater, but relatively close in value, to the mass of a hydrogen ion or proton whose mass is  $1.6667 \times 10^{-27} \text{ kg}$ . It was noted earlier that protons play a central role in the synthesis of ATP, the energy molecule that enables most biological functions. It is thus possible to view  $dM$  as the average mass of metabolism particles, such as protons or hydrogen ions  $H^+$  and other particles, mostly heavier ones, that enable the mitochondrial ATP synthesis. Moreover, the ‘lower’ adult lifespan achieved with a ‘smaller’  $dM$  value seen from Table IV supports the theory that more free radicals are created for this case. This would be the case since more high energy electrons must be used to produce an increased number of protons for ATP synthesis. This result is thus consistent with the free radical theory of aging where the creation of free radicals such as superoxide  $O_2^-$  increases the number of mitochondrial-DNA mutations, thus resulting in faster aging. In conclusion, our LTT thermal physics theoretical results as well as the aging agent behavior of free radical creation in the mitochondria leads us to conclude that an increased CNR per thermote rate should lead to a longer adult lifespan. Thus, our LTT adult lifespan bounds results can be said to reasonably and sensibly support Harman’s free radical theory of aging!

**Table I. The Macro Nutritional Consumption Rate (NCR) Lifespan Eq. Results for Three Body Masses**

Theoretical Adult Lifespan $\tau$ plus 18 years of Childhood	Nutritional Consumption Rate or NCR $\Delta M$ ( QoO Heat Energy $\delta Q$ in MJ, Pace of Lifebits $II$ )		
	M = 50 kg $v = 1.2083 \times 10^{-4} \text{ m/sec}$	M = 70 kg $v = 1.3518 \times 10^{-4} \text{ m/sec}$	M = 100 kg $v = 1.5224 \times 10^{-4} \text{ m/sec}$
102 + 18 = 120 yrs	0.2591 kg or 1,296 kcal ( 5.41 MJ, 2040 yrs/m <sup>3</sup> )	0.3628 kg or 1,814 kcal ( 7.58 MJ, 1457 yrs/m <sup>3</sup> )	0.5183 kg or 2,592 kcal ( 10.83 MJ, 1020 yrs/m <sup>3</sup> )
82 + 18 = 100 yrs	0.2889 kg or 1,445 kcal ( 6.04 MJ, 1640 yrs/m <sup>3</sup> )	0.4048 kg or 2,024 kcal ( 8.46 MJ, 1171 yrs/m <sup>3</sup> )	0.5780 kg or 2,890 kcal ( 12.08 MJ, 820 yrs/m <sup>3</sup> )
62 + 18 = 80 yrs	0.3324 kg or 1,662 kcal ( 6.95 MJ, 1240 yrs/m <sup>3</sup> )	0.4653 kg or 2,327 kcal ( 9.73 MJ, 886 yrs/m <sup>3</sup> )	0.6647 kg or 3,324 kcal ( 13.89 MJ, 620 yrs/m <sup>3</sup> )
42 + 18 = 60 yrs	0.4038 kg or 2,019 kcal ( 8.44 MJ, 840 yrs/m <sup>3</sup> )	0.5654 kg or 2,827 kcal ( 11.82 MJ, 600 yrs/m <sup>3</sup> )	0.8077 kg or 4,039 kcal ( 16.88 MJ, 420 yrs/m <sup>3</sup> )

**Table II. The Micro DoF Lifespan Eq. Results for Three Body Masse**

Theoretical Adult Lifespan $\tau$ plus 18 years of Childhood	DoF Heat-Capacity $c_h(\eta)$ where $c_{h,Max} = 5/2 = 2.5$		
	M = 50 kg with $\eta_{Max} = 0.8471108$	M = 70 kg with $\eta_{Max} = 0.8476936$	M = 100 kg with $\eta_{Max} = 0.8483065$
102 + 18 = 120 yrs	2.3403	2.3410	2.3418
82 + 18 = 100 yrs	2.3433	2.3440	2.3448
62 + 18 = 80 yrs	2.3471	2.3478	2.3485
42 + 18 = 60 yrs	2.3525	2.3531	2.3538

**Table III. The Micro-Macro DoF-NCR Lifespan Eq. Results for Three Body Masses**

Theoretical Adult Lifespan $\tau$ plus 18 years of Childhood	Heat Energy Rate $\delta Q$ (Gibbs $dG$ % of $\delta Q$ , Enthalpy $dH$ % of $\delta Q$ , Internal Energy $dE$ % of $\delta Q$ , Work $dW = PdV$ % of $\delta Q$ )		
	M = 50 kg Number of Water Molecules is $M/m = 1.6666 \times 10^{27}$	M = 70 kg Number of Water Molecules is $M/m = 2.3333 \times 10^{27}$	M = 100 kg Number of Water Molecules is $M/m = 3.3333 \times 10^{27}$
102 + 18 = 120 yrs	5.41 MJ (69.1 %, 30.9 %, 21.65 %, 9.25 %)	7.58 MJ (69.1 %, 30.9 %, 21.65 %, 9.25 %)	10.83 MJ (69.1 %, 30.9 %, 21.65 %, 9.25 %)
82 + 18 = 100 yrs	6.04 MJ (68.4 %, 31.6 %, 22.12 %, 9.48 %)	8.46 MJ (68.4 %, 31.6 %, 22.12 %, 9.48 %)	12.08 MJ (68.4 %, 31.6 %, 22.12 %, 9.48 %)
62 + 18 = 80 yrs	6.95 MJ (67.6 %, 32.4 %, 22.72 %, 9.68 %)	9.73 MJ (67.6 %, 32.4 %, 22.72 %, 9.68 %)	13.89 MJ (67.6 %, 32.4 %, 22.72 %, 9.68 %)
42 + 18 = 60 yrs	8.44 MJ (66.3 %, 33.7 %, 23.65 %, 10.05 %)	11.82 MJ (66.3 %, 33.7 %, 23.65 %, 10.05 %)	16.88 MJ (66.3 %, 33.7 %, 23.65 %, 10.05 %)

**Table IV. The Micro-Macro DoF-NCR Lifespan Eq. Results for Three Body Masses (Continued)**

Theoretical Adult Lifespan $\tau$ plus 18 years of Childhood	NCR per Thermote Rate, $dM = \Delta M/dJ$ , Followed by $B(\eta)$ Number ( Thermote Rate $dJ$ , Specific Heat-Capacity $C_h(\eta)$ )		
	M = 50 kg Number of Water Molecules is $M/m = 1.6666 \times 10^{27}$	M = 70 kg Number of Water Molecules is $M/m = 2.3333 \times 10^{27}$	M = 100 kg Number of Water Molecules is $M/m = 3.3333 \times 10^{27}$
102 + 18 = 120 yrs	2.2144 x 10 <sup>-27</sup> kg, 1.3342 (1.1702 x 10 <sup>26</sup> , 3303 J/kg.K.DoF)	2.2144 x 10 <sup>-27</sup> kg, 1.3332 (1.6363 x 10 <sup>26</sup> , 3300 J/kg.K.DoF)	2.2144 x 10 <sup>-27</sup> kg, 1.3324 (2.342 x 10 <sup>26</sup> , 3306 J/kg.K.DoF)
82 + 18 = 100 yrs	2.1692 x 10 <sup>-27</sup> kg, 1.3305 (1.3324 x 10 <sup>26</sup> , 3374 J/kg.K.DoF)	2.1692 x 10 <sup>-27</sup> kg, 1.3297 (1.8642 x 10 <sup>26</sup> , 3373 J/kg.K.DoF)	2.1692 x 10 <sup>-27</sup> kg, 1.3290 (2.663 x 10 <sup>26</sup> , 3375 J/kg.K.DoF)
62 + 18 = 80 yrs	2.1113 x 10 <sup>-27</sup> kg, 1.3264 (1.5720 x 10 <sup>26</sup> , 3470 J/kg.K.DoF)	2.1113 x 10 <sup>-27</sup> kg, 1.3255 (2.2007 x 10 <sup>26</sup> , 3470 J/kg.K.DoF)	2.1113 x 10 <sup>-27</sup> kg, 1.3248 (3.1435 x 10 <sup>26</sup> , 3470 J/kg.K.DoF)
42 + 18 = 60 yrs	2.0308 x 10 <sup>-27</sup> kg, 1.3202 (1.9892 x 10 <sup>26</sup> , 3622 J/kg.K.DoF)	2.0308 x 10 <sup>-27</sup> kg, 1.3196 (2.777 x 10 <sup>26</sup> , 3612 J/kg.K.DoF)	2.0308 x 10 <sup>-27</sup> kg, 1.3188 (3.965 x 10 <sup>26</sup> , 3611 J/kg.K.DoF)

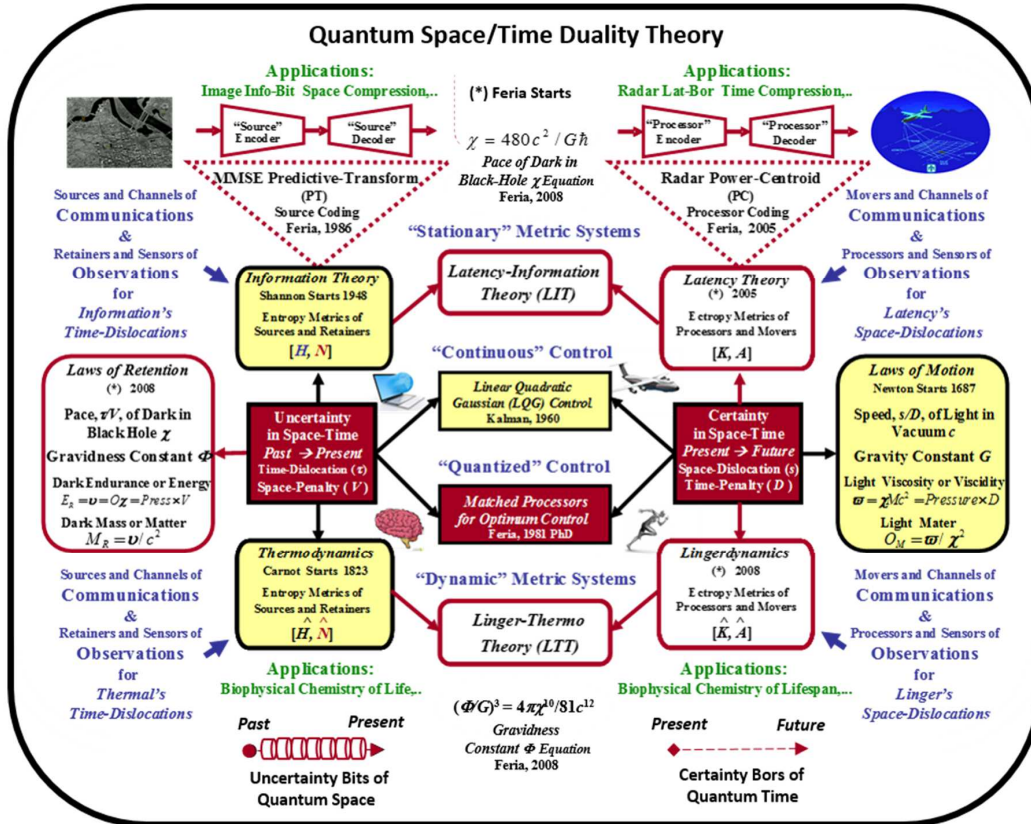


Fig. 4. The Timeline of Development of the Universal Cybernetics Duality Principle

## V. CONCLUSIONS

In this paper a novel universal DoF-TSE equation or UDTE was revealed which offered a simple, new, method for finding the thermal source entropy or TSE of a homogeneous medium. The scheme expressed the TSE as the ratio of the medium's mass-energy over the product of the number of its degrees of freedom times the kinetic thermal energy contributed by each. Thus revealing an 'inverse' relationship between the medium's TSE and temperature. The mediums to which the UDTE was applied were a black-hole, a photon gas and a flexible phase, that span applications from astrophysics to biological lifespan.

The roots of the UDTE were traced first. They were found in linger thermo theory or LTT, an emerging theory in thermal physics. The LTT, in turn, was noted to have surfaced from the universal cybernetics duality principle or UCDP, a quantum space/time duality theory, whose timeline of development is shown in Fig. 4 for ease of recall in applications. Universal cybernetics studies control and communication issues in living and non-living systems and its study was started by Norbert Wiener in 1948 with his book entitled "Cybernetics or Control and Communication in the Animal and the Machine." The blocks in Fig. 4 list major scientific approaches from the past that are the pillars of the UCDP. These are: the Laws of

Motion in Physics started in 1687 by Newton; Thermodynamics started in 1823 by Carnot; and Information Theory started in 1948 by Shannon. The UCDP was first identified in 1978 in Kalman LQG 'continuous' control as part of graduate studies in three cybernetics areas. They were communications, biofeedback and optimum control that eventually led to the 1981 PhD dissertation "Matched Processors for Optimum Control", also noted in the figure. Since then Latency Theory, the Laws of Retention in Physics, and Lingerdynamics have inherently surfaced from the UCDP as is seen in Fig. 4. Finally the UDTE is found in LTT, which combines thermodynamics and lingerdynamics as is displayed on the bottom of Fig. 4.

The new advanced UDTE had an immediate application in astrophysics where the TSE of a black-hole was now simply found. The scheme was also successfully applied to the TSE of a flexible phase water medium used to derive sensible theoretical lifespan bounds for organisms. Two bounds were offered. One was a DoF based bound that expressed theoretical biological lifespan as a function of the medium's DoF heat capacity. The second was a mixed nutritional consumption rate (NCR)-DoF bound whose expressions outwardly revealed an inverse relation between lifespan and the number of protons created. This LTT revelation backed the Harman free radicals theory of aging that suggests mitochondria



DNA mutations are responsible for an organism's aging. These mutations were elicited by free radicals produced when protons were created by high energy electrons to power the ATP Synthase in its synthesis of ATP, the cell's energy molecule.

Another important disclosure of this paper was that the inverse relationship between the medium's TSE and temperature seen in the UDTE was in perfect agreement with the LTT revelation of 2011 [35] that the 2<sup>nd</sup> law of thermodynamics should be enhanced with three additional 2<sup>nd</sup> laws. This enhancement reflected then a continuous decrease in the retention, processing and motion power of a medium. When these laws were viewed jointly they led us to a novel description for the behavior of mediums. This description was that with the passing of time closed homogeneous mediums would evolve not only towards disorder but also towards release, disconnection and immobility, thus further emphasizing their "aging" or "arrow of time" behavior. In particular, the consistency of the aforementioned 2<sup>nd</sup> laws with the inverse relationship between the TSE and temperature in the UDTE was simply revealed when the 2<sup>nd</sup> law of retention thermodynamics was noted to require the medium surface area or volume to continuously increase, which in turn implied the lowering of the medium temperature as the TSE increased with the passing of time.

A final reflection on our application of the revealed universal DoF-TSE equation of this paper to astrophysics and human lifespan cybernetic problems is that its UCDP anchor, similarly as it did for high-performance radar [27], has sensibly guided us to surprisingly efficient as well as affordable analysis and design formulations. These UCDP formulations have exploited Nature's innate quantum space/time dualities, to yield sensible results that exult in us great hope for future UCDP driven cybernetic solutions in numerous research fields.

*This article is dedicated to the memory of Rudolf E. Kalman (1930-2016), whose contributions to control and estimation theory have greatly impacted modern control engineering as well as the author's research. This article is also dedicated to the memory of Denham Harman (1916-2014), the father of the free radical theory of aging.*

## APPENDIX A

### Pace of Dark in a Black-Hole

In this appendix we derive the expression for the pace of an uncharged, and non-rotating black-hole given by:

$$\Pi = \frac{\tau}{V} = \chi = \frac{480c^2}{\hbar G} = 6.1203 \times 10^{63} \text{ sec}/m^3 \quad (110)$$

that was first derived in [21]. This pace goes by the name 'the pace of dark in a black-hole' and has the assigned symbol  $\chi$ . The variables in (110) are defined as follows:  $c$  is the speed of light in a vacuum (which is the motion dual of the pace of dark  $\chi$  for black-hole retention);  $\hbar$  is the reduced Planck constant;  $G$  is the gravitational constant;  $\tau$  is the time-dislocation (or lifespan) of the black-hole; and the shape of the black-hole is spherical of radius  $r$  and volume  $V$  expression given by  $V=4\pi r^3/3$ .

The derivation starts with the assumption that the black-hole satisfies the black body luminance ( $dE/dt$ ) expression [2] according to:

$$-\frac{dE}{dt} = \frac{\pi^2}{60\hbar^3 c^2} (4\pi r^2)(kT)^4 \quad (111)$$

where  $k$  is the Boltzmann constant,  $T$  is temperature,  $kT$  is thermal energy, and  $E=Mc^2$  is the mass-energy of the black-hole. Next the Schwarzschild radius expression for the event horizon of a black-hole is noted to be given by:

$$r = \frac{2GM}{c^2} = \frac{2GE}{c^4} \quad (112)$$

Next the Bekenstein-Hawking entropy ( $S$ ) for our spherical black-hole [2] is:

$$S = \frac{kc^3}{4\hbar G} 4\pi r^2 = \frac{kc^3}{4\hbar G} 4\pi \left( \frac{2GE}{c^4} \right)^2 \quad (113)$$

where the expression (112) has been used for  $r$ . Next the following thermal-energy expression is derived via (113):

$$kT = (\partial S / \partial E)^{-1} = \frac{\hbar c^5}{8\pi GE} \quad (114)$$

Next making use of (114) in (111) one obtains the luminance non-linear differential equation:

$$-\frac{dE}{dt} = \frac{\hbar c^{10}}{15360\pi G^2 E^2} \quad (115)$$

Solving (115) one then finds the following energy expression for the black-hole at time  $t$ :

$$E^3(t) = E^3(t)|_{t=0} - \frac{\hbar c^{10}}{5120\pi G^2} t = E^3 - \frac{\hbar c^{10}}{5120\pi G^2} t \quad (116)$$

Next the lifespan  $\tau$  of the black-hole is found by setting  $E(t)$  equal to zero to yield:

$$t|_{E(t)=0} = \tau = \frac{5120\pi G^2}{\hbar c^{10}} E^3 \quad (117)$$

Finally using (112) and  $V=4\pi r^3/3$  in (117) the desired result (110) is derived.

## APPENDIX B

### The Gravidness Constant

In this appendix we derive the gravidness constant  $\Phi$  expression given by:

$$\Phi = \sqrt[3]{4\pi\chi^{10} / 81c^{12}} G = 1.8619 \times 10^{168} \text{ Pa} \cdot \text{sec}^{4/3} / (N \cdot m^6 / \text{sec})^2 \quad (118)$$

that was first derived in [21] where:  $G$  is the gravitational constant;  $c$  is the speed of light in a vacuum; and  $\chi$  is the pace of dark in a black-hole.

The derivation starts with the gravitational force  $f^{M_R \leftarrow m_R}$  which is the attraction force exerted by the point-mass  $M_R$  (viewed as a retention dark mass) at the center of a

spherical black-hole, of radius  $r$ , that acts on another point-mass  $m_R$  at the radial distance  $r$  according to:

$$f^{M_R \leftarrow m_R} = GM_R m_R / r^2 \quad (119)$$

Next the retention press  $\gamma^{O \leftarrow o}$  (or pressure), which is the attraction pressure exerted by the point-mater  $O$  (the retention dual of the mass  $M_R$  in energy per pace units) that acts on another point-mater  $o$  (the retention dual of  $m_R$ ), is found by dividing (119) by the surface area  $4\pi r^2$  of the black-hole sphere, see bottom of Fig. 2, to yield

$$\gamma^{O \leftarrow o} = \frac{f^{M_R \leftarrow m_R}}{4\pi r^2} = \frac{Gm_R M_R}{4\pi r^2 r^2} = \frac{Gm_R M_R}{(4\pi^3/3)(3r)} = \frac{Gm_R M_R}{V \times 3r} \quad (120)$$

Next using  $\chi = \tau/V$  in (120) one obtains:

$$\gamma^{O \leftarrow o} = \frac{Gm_R M_R}{V \times 3r} = \frac{Gm_R M_R}{(\tau/\chi) \times \sqrt[3]{81\tau/4\pi\chi}} \quad (121)$$

Next simplifying (121) one finds:

$$\gamma^{O \leftarrow o} = \sqrt[3]{\frac{4\pi\chi^4}{81}} G \frac{m_R M_R}{\tau^{4/3}} \quad (122)$$

Next substituting in the endurance equation  $\nu = O\Pi$  (the retention dual of momentum) the dark energy of retention  $E_R = M_R c^2$  for the endurance  $\nu$  and the pace of dark  $\chi$  for the pace  $\Pi$  one finds:

$$\nu = O\Pi = E_R = M_R c^2 = O\chi \quad (123)$$

Using (123), including  $m_R c^2 = O\chi$ , in (122) one derives:

$$\gamma^{O \leftarrow o} = \frac{\sqrt[3]{4\pi\chi^{10}/81c^{12}} G pO}{\tau^{4/3}} = \frac{\Phi o O}{\tau^{4/3}} \quad (124)$$

which results in (118) as desired.

## REFERENCES

- [1] P. Atkin, *Four Laws that Drive the Universe*, Oxford University Press, 2007
- [2] S. Lloyd, "Ultimate physical limits to computation", *Nature*, Aug. 2000
- [3] Black-body radiation—*Wikipedia*, the free encyclopedia
- [4] E.H. Feria, "On the universe's cybernetics duality behavior", <http://feria.csi.cuny.edu>, *Proc. of SPIE*, vol. 9497, pp. 1-20, Apr. 2015
- [5] E.H. Feria, "The flexible-phase entropy and its rise from the universal cybernetics duality", *IEEE Int'l Conf. on Systems, Man and Cybernetics*, San Diego, CA, USA, 5-8 October 2014
- [6] R.A. Freitas Jr., *Nanomedicine, Vol. I: Basic Capabilities*, Landes Bioscience, 1999
- [7] D. Harman, "The aging process", *Proceedings of the National Academy of Sciences* 78 (24): 7124-8., 1981
- [8] V.L. Bengtson, D. Gans, N.M. Putney, and M. Silverstein, *Handbook of Theories of Aging*, Second Ed., Springer Publ. Comp., 2008
- [9] P. Vitello, "Denham Harman, 98, Dies; Sought Leverage on Aging", *New York Times*, November 28, 2014
- [10] M.H. Jones, R.J. Lambourne, *An Introduction to Galaxies and Cosmology*, Cambridge University Press, 2004.
- [11] N. Wiener, *Cybernetics or Control and Communication in the Animal and the Machine*, MIT Press, 1948
- [12] M. Athans, "The Role and Use of the Stochastic Linear Quadratic Gaussian Problem in Control System Design". *IEEE Transaction on Automatic Control*, AC-16: pp. 529-552, 1971
- [13] Remembering Rudolf E. Kalman, 1930-2016, Herbert Wertheim College of Engineering, Gainesville, University of Florida, <https://www.eng.ufl.edu/news/remembering-rudolf-e-kalman-1930-2016/>
- [14] E.H. Feria, *Matched Processors for Optimum Control*, Ph.D. Dissertation, CUNY Graduate Center, August 1981
- [15] E.H. Feria, "Mathematical model for central nervous system (CNS) mechanism that control movements," *New York University Medical Center Grant*, RF-CUNY, 1981
- [16] S. Finger, *Origins of Neuroscience: A History of Explorations into Brain Function*. Oxford University Press, 2004
- [17] E.H. Feria, "Matched processors for quantized control: A practical parallel processing approach", *International Journal of Controls*, vol. 42, issue 3, pp. 695-713, September, 1985
- [18] J.M., Wozencraft and I.M., Jacobs, *Principles of Communication Engineering*, Waveland Press, Inc., 1965
- [19] E.H. Feria, "Latency information theory: A novel latency theory revealed as time dual of information theory", *Proc. of IEEE Signal Processing and Education Workshop 5*, pp. 107-112, Jan. 2009
- [20] C.E. Shannon, "A Mathematical Theory of Communication", *Bell System Tech. Journal*, vol. 27, pp. 379-423, 623-656, July, Oct., 1948
- [21] E.H. Feria, "Latency information theory and applications: Part III. On the discovery of the space-dual of the laws of motion in physics," *Proc. of SPIE*, vol. 6982, pp. 698212(1-16), March 2008
- [22] E.H. Feria, "Latency information theory: The mathematical-physical theory of communication-observation", *Proc. of IEEE Sarnoff Symposium*, Princeton, N.J., 8 pp., April 2010
- [23] E.H. Feria, "The Latency Information Theory Revolution, Part II: Its Statistical Physics Bridges and the Discovery of the Time Dual of Thermodynamics", *Proceedings of SPIE*, Vol 7708-30, pp. 1-22, Orlando, FL, USA, 5-9 April 2010
- [24] A.G. Riess, et al., "Type Ia Supernova Discoveries at  $z > 1$  from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution," *Astrophysical Journal*, 2004
- [25] E.H. Feria, "A predictive transform compression architecture and methodology for KASSPER", *Final Technical Report, DARPA Grant FA8750-04-1-0047*, May 2006
- [26] E.H. Feria, "Methods and applications utilizing signal source memory space compression and signal processor computational time compression," *US Patent 7773032*, 2010, *US Patent 8098196*, 2012
- [27] E.H. Feria, "Maximizing the efficiency and affordability of high-performance radar", <http://dx.doi.org/10.1117/2.1201407.005429> or <http://feria.csi.cuny.edu>, *SPIE Newsroom*, July 2014
- [28] L.E. Romans, *Computed Tomography for Technologists: A Comprehensive Text*, Walters Kluwer Health, Lippincott Williams & Wilkins, 2011
- [29] E.H. Feria, "Predictive Transform Coding", <http://feria.csi.cuny.edu> *IEEE NAECON*, Vol. I, pp. 46-52, Dayton, Ohio, May 1986
- [30] E.H. Feria, "Linear predictive transform of monochrome images," <http://feria.csi.cuny.edu>, *Image and Vision Computing*, London: Butterworth, pp. 267-279, Nov. 1987
- [31] E.H. Feria, "Predictive Transform Estimation", *IEEE Transactions on Signal Processing*, pp. 2481-2499, Vol. 39, No. 11, Nov. 1991
- [32] E.H. Feria, "Linear predictive transform: A unified approach to signal modeling, coding, estimation and control," *IEEE DSP Workshop*, South Lake Tahoe, CA, Sept. 1988, "Predictive transform: Signal coding and modeling, Part I" *11th World Congress IFAC*, Tallinn, Estonia, Aug. 1990, "Predictive transform detection: A unifying framework for source coding, channel modeling, and detection," in *IEEE DSP Workshop*, Mohonk, NY, Sept. 1990
- [33] E.H. Feria, "Optimum Channel and Source Integrated Coding", *IEEE NAECON*, Vol. I, pp. 498-506, Dayton, Ohio, May 1994
- [34] E.H. Feria, "Predictive Transform Source Coding with Subbands", *US Patent, 8,150,183*, 2012, *US Patent, 8,428,376 B2*, 2013
- [35] E.H. Feria, "Latency information theory: Novel linderdynamics ectropies are revealed as time duals of thermodynamics entropies", *IEEE Int'l Conf. on Systems, Man and Cybernetics*, 8 pages, Anchorage, Alaska, USA, Oct. 2011
- [36] A.H. Carter, *Classical and Statistical Thermodynamics*, Pr. Hall, 2001
- [37] M.M. Mano and M.D. Ciletti, *Digital Design*, Prentice Hall, 2013
- [38] X. Dong, B. Milholland, J. Vijg, "Evidence for a limit to human lifespan", *Nature*, Oct. 5, 2016
- [39] C. Zimmer, "Human Life Span Reaches Ceiling at 115, a Study Says", *New York Times*, Oct. 6, 2016
- [40] P. Paulev, *Medical Physiology and Pathophysiology*, 2<sup>nd</sup> Edition, Chapter 21, Copenhagen Medical Publishers, 1999-2000