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PREDICTIVE TRANSFORM CODING

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ABSTRACT

In this paper a novel predictive transform coding algorithm is offered for the bandwidth compression of digital data. The algorithm uses a subset of previously estimated data samples to linearly predict the coefficients resulting from the transformation of a data block. The algorithm transform and predictor matrices are derived by minimizing the mean square error between a data block and its estimate subject to two constraints. These constraints are that the coefficient errors, i.e. the difference between the transform coefficients and their estimates, must be uncorrelated and also have zero mean value. These constraints allow us to encode the coefficient errors using easily implementable unbiased scalar quantizers. It is verified via an illustrative video application that the predictive transform algorithm integrates the ease of implementation of predictive coding (12), (1), with the high reproduction fidelity of transform coding (11), (15) into a general and quite practical bandwidth compression technique.

I. INTRODUCTION

Predictive transform coding arises from the desire to obtain with a readily implementable coder substantial bandwidth compression while still deriving a high reproduction fidelity of the processed data. The key property of a predictive transform algorithm is that the coefficients resulting from the transformation of a data block are predicted using a subset of previously estimated data samples. The specific characteristics of our algorithm that result in a readily implementable coder with low reproduction distortion at high compression rates are several. These are: 1. The subset of previously estimated samples that are used to predict each transform coefficient is composed of only those samples which are significantly correlated to the transform coefficients. 2. In practical applications unbiased scalar quantizers can be used to encode the coefficient errors between the transform coefficients and their predicted values. This is the case since the algorithm transform and predictor matrices minimize the mean square error between a data block and its estimate subject to the constraint that the coefficient errors are uncorrelated and also have zero mean value. Although scalar quantizers are only strictly optimum in the case where the coefficient errors are also independent, e.g. for uncorrelated gaussian data, it is often found that scalar quantizers are quite adequate in applications dealing with uncorrelated but dependent data samples (see (5) and section IV).

3. In practical applications such as the video example of section IV it is found that the multiplications required by the transform and predictor matrices can be easily implemented with additions, subtractions, and register shifts. This is due to the robustness of the predictor and transform matrices which yield excellent performance even when their optimum elements are roughly approximated by readily implementable values such as 0, 1/2, 1/4, 3/4, etc.

4. Relatively small data blocks can yield low reproduction distortion. This is again illustrated with our video example and originates from our prediction of the transform coefficients via the past data estimates that are most highly correlated to the coefficient errors.

One fundamental byproduct of the general nature of our algorithm is that it serves as a natural bridge between two coding schemes, i.e., predictive coding and transform coding (5). We then have that our algorithm serves the dual purpose of providing a performance upperbound for both predictive coding and transform coding.

For a comprehensive survey of the application of predictive coding and transform coding to the bandwidth compression of digital data the reader is referred to the recent book by Jayant and Noll (5).

II. CODER STRUCTURE

We now present the general structure of our predictive transform coder as it relates to a two-dimensional data block. The reason for using a two-dimensional data block is twofold. First it clearly illustrates the general applicability of the scheme and second it is used in the illustrative video example of section IV.

The two-dimensional data upon which we apply the predictive transform algorithm is illustrated in Fig. 1. Note that each data block is represented by the vector

$$x(i) = [x_1(i),..,x_N(i),..,x_W(i)]^t \text{ for all } i$$

where the index i denotes the time step at which the block was generated, $x_j(i)$ denotes the j-th data point in the i-th block, N is the block size, and W is the number of elements in the data block.

The structure of the proposed predictive

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transform coder is shown in figure 2 where each major block in the coder is defined next.

Transform:

$$c(k+1) = T^{-1}x(k+1) \qquad (1)$$

or

$$x(k+1) = Tc(k+1) = \sum_{i=1}^{W} T_i c_i(k+1) \qquad (2)$$

where: a) T is a WxW transform matrix with $W=N^2$; b) c(k+1) is a coefficient vector with W components, i.e.

$$c(k+1) = [c_1(k+1),\ldots,c_W(k+1)]^t; \qquad (3)$$

and c) T_i is the i-th column of T as defined in figure 2.

Estimator:

$$\hat{x}(k+1) = T\hat{c}(k+1) \qquad (4)$$

$$\hat{c}(k+1) = \hat{c}'(k) + \Delta\hat{c}(k) \qquad (5)$$

where: a) $\hat{x}(k+1)$ is the estimate of the data block x(k+1); b) $\hat{c}(k+1)$ is an estimate of the coefficient vector c(k+1); c) $\hat{c}'(k)$ is a 'preliminary' estimate of the coefficient vector c(k+1); & d) $\Delta\hat{c}(k)$ is the quantized coefficient error.

Predictor:

$$\hat{c}'(k) = P^t z(k) = \begin{bmatrix} P_1^t \\ \vdots \\ P_W^t \end{bmatrix} z(k) \qquad (6)$$

where: a) P^t is a WxM predictor matrix also defined in Fig. 2; b) z(k) is a M-dimensional vector whose elements can be any subset of all previously estimated data elements. For example in the video illustrative example of section IV the following subsets of past estimated data elements will be used:

Case 1: In this case it is assumed that we operate on 1x1 blocks and use immediately adjacent past data elements, i.e.,

$$z(k) = [\hat{x}(k-L),\hat{x}(k+1-L),\hat{x}(k+2-L),\hat{x}(k)]^t. \qquad (7)$$

Case 2: We operate on 2x2 blocks and use immediately adjacent past data elements, i.e.,

$$z(k) = [\hat{x}_4(k-L),\hat{x}_3(k+1-L),\hat{x}_4(k+1-L),\hat{x}_3(k+2-L),\hat{x}_2(k),\hat{x}_4(k)]^t. \qquad (8)$$

Case 3: 4x4 blocks are used with immediately adjacent past data elements, i.e.,

$$z(k) = [\hat{x}_{16}(k-L),\hat{x}_{13}(k+1-L),\hat{x}_{14}(k+1-L),\hat{x}_{15}(k+1-L),\hat{x}_{16}(k+1-L),\hat{x}_{13}(k+2-L),\hat{x}_4(k),\hat{x}_8(k),\hat{x}_{12}(k),\hat{x}_{16}(k)]^t; \qquad (9)$$

and c) P_i^t is the i-th row of P^t which when multiplied by the vector z(k) yields a predicted estimate of the i-th coefficient $c_i(k+1)$.

Quantizer:

$$\Delta\hat{c}(k) = Q(\Delta c(k)) \qquad (10)$$

where the operator Q(.) represents the vector quantization of the coefficient vector error $\Delta c(k)$.

III. Optimum Predictive Transform Coder

In this section optimum transform and predictor matrices are derived for our coder under several constraints. We first present these constraints, then make a mathematical statement of the problem, and finally present its solution.

Constraints

There are four constraints. These are:

Constraint 1: The basis vectors $\{T_i\}$ of the transform T will be constrained to be orthonomal, i.e.,

$$T_i^t T_j = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \qquad (13)$$

One reason for this constraint is to give equal weight to the energy associated with each coefficient error. Note that this constraint also implies that T is an unitary matrix, i.e.,

$$T^{-1} = T^t. \qquad (14)$$

A second reason for this constraint is that it results in uncorrelated coefficient errors as shown in appendix A. This in turn implies that in applications (see Section IV) we can use simple scalar quantizers to encode each coefficient error, i.e.,

$$Q(\Delta c(k)) = [Q_1(\Delta c_1(k)),\ldots,Q_W(\Delta c_W(k))]^t$$

where $Q_j(\Delta c_j(k))$ represents the scalar quantization of the coefficient error $\Delta c_j(k)$. Note that the scalar quantizers are not generally optimum since the coefficient errors often remain statistically dependent even if they are uncorrelated.

Constraint 2: The optimum transform and predictor matrices must yield coefficient error components with zero mean value, i.e.,

$$E[\Delta c_i(k)] = 0 \quad \text{for all } i. \qquad (15)$$

The objective of this constraint is to simplify the design of the scalar quantizer since it then follows that we do not need to be concerned about coefficient error elements with a nonzero mean component. It should be noted that the constraints (15) and (14) further imply the following constraint on the transform and predictor matrices:

$$U_W^t T_i - U_M^t P_i = 0 \text{ for all } i \qquad (16)$$

where U_W and U_M are unit column vectors with W and M elements, respectively. This constraint can be readily derived as follows:

First, using Fig. 2 and Eq. (14) we note that

$$\Delta c_i(k) = T_i^t x(k+1) - P_i^t z(k) \text{ for all } i. \qquad (17)$$

Second and last, taking the expected value of Eq. (17), using constraint (15), and assuming that the expected value of each data sample is constant we obtain the desired result (16).

Constraint 3: The quantizer will be assumed to work as follows: a) J arbitrary coefficient error components are unaffected by the quantizer, i.e.,

$$\Delta\hat{c}_i(k) = \Delta c_i(k) \qquad (18)$$

for J arbitrary components of $\Delta c(k)$ where $J < W$; and b) the remaining W-J coefficient error components are set to zero by the quantizer, i.e.,

$$\Delta\hat{c}_i(k) = 0 \qquad (19)$$

for the remaining W-J components of $\Delta\hat{c}(k)$. The basic advantage of this constraint is that it makes the evaluation of the transform and predictor matrices a mathematically tractable problem.

Constraint 4: The mean square error (MSE)

$$E\left[(x(k+1)-\hat{x}(k+1))^t(x(k+1)-\hat{x}(k+1))\right] \quad (20)$$

between the current picture block x(k+1) and its estimate $\hat{x}(k+1)$ will be used as the performance criterion that the transform and predictor matrix should minimize. The selection of this criterion mainly arises due to its well known mathematical tractability properties.

Statement of Problem

The statement of the coder design problem is as follows:

Given the previously stated constraints (13)-(19) determine the transform and predictor matrices T and P that minimize the MSE (20).

Solution:

The solution to the previously stated coder design problem is now given in the form of a theorem whose proof is given in appendix A.

Theorem 1: The transform and predictor matrices T and P that minimize the MSE (20) subject to constraints (13)-(19) are obtained from the evaluation of the following matrix equations:

$$\left\{E\left[x(k+1)x^t(k+1)\right] - A\right\}T_i = \lambda_i T_i \quad i=1,\ldots,W \quad (21)$$

$$A=\begin{bmatrix} E\left[x(k+1)z^t(k)\right] & \begin{matrix}1/2\\ \vdots \\ 1/2\end{matrix}\end{bmatrix}\begin{bmatrix} E\left[z(k)z^t(k)\right] & \begin{matrix}1/2\\ \vdots\\ 1/2\end{matrix} \\ 1/2\ldots\ldots1/2 & 0\end{bmatrix}^{-1}\cdot\begin{bmatrix} E\left[z(k)x^t(k+1)\right] \\ 1/2\ldots\ldots\ldots1/2\end{bmatrix} \quad (22)$$

and

$$\begin{bmatrix} P_i \\ \mu_i \end{bmatrix} = \begin{bmatrix} E\left[z(k)z^t(k)\right] & \begin{matrix}1/2\\ \vdots\\ 1/2\end{matrix} \\ 1/2\ldots\ldots1/2 & 0\end{bmatrix}^{-1}\begin{bmatrix} E\left[z(k)x^t(k+1)\right] \\ 1/2\ldots\ldots\ldots1/2\end{bmatrix}T_i$$

$$i = 1,\ldots,W \quad (23)$$

where: a) $\{T_i\}$ and $\{P_i\}$ are the columns of the transform and predictor matrices T and P; b) $E\left[x(k+1)x^t(k+1)\right]$ is the second moment statistic matrix of the k-th data block x(k+1); c) $E\left[z(k)z^t(k)\right]$ is the second moment statistic of the data elements z(k) surrounding the block x(k+1); d) $E\left[z(k)x^t(k+1)\right]$ is the correlation matrix between the block x(k+1) and its surrounding data elements z(k); e) λ_i is a Lagrange multiplier associated with the constraint $T_i^tT_i = 1$ for all i; f) μ_i is a Lagrange multiplier associated with the mean constraint (16); and g) the matrix inversion shown in eqs. (22) and (23) is assumed to exist.

In addition, the minimum MSE obtained with these matrices is given by

$$\min_{T,P} E\left[(x(k+1)-\hat{x}(k+1))^t(x(k+1)-\hat{x}(k+1))\right]$$

$$= \sum_{i=J+1}^{W} E\left[\Delta c_i^2(k)\right]$$

$$= \sum_{i=J+1}^{W} T_i^t\left\{E\left[x(k+1)x^t(k+1)\right] - A\right\}T_i$$

$$= \sum_{i=J+1}^{W} \lambda_i \quad (24)$$

where $\lambda_{J+1},\ldots,\lambda_W$ are the smallest W-J eigenvalues of the eigensystem (21).

Also, the optimum transform and predictor matrices result in uncorrelated coefficient errors.

IV. A Video Processing Example

In this section we apply predictive transform coding to the interfield coding of a monochrome image. In addition, we contrast the complexity and performance of our coders to that of purely predictive and purely transform coders.

The coders that will be used in our study are the following:

Coder 1. K-L transform 2x2 coder. This coder is the best purely transform coder that can be obtained with a 2x2 data block (5), (7), (9).

Coder 2. K-L transform 4x4 coder. Again this coder is the best purely transform coder that can be obtained with a 4x4 data block.

Coder 3. Predictive transform 1x1 coder using immediately adjacent past picture elements, i.e., equation (7). It should be noted that this coder is similar to that of an optimum two-dimensional purely predictive coder (8).

Coder 4. Predictive transform 2x2 coder using immediately adjacent past picture elements, i.e., equation (8).

Coder 5. Predictive transform 4x4 coder using immediately adjacent past picture elements, i.e., equation (9).

Coder 6. Simple predictive transform 2x2 coder using immediately adjacent past picture elements and with predictor and transform matrices that can be readily implemented.

Data Base: The two black and white images shown in figure 3 are used to generate the second order statistics needed in the evaluation of T and P. The images are sampled at 450 samples per line (7.2 MHz sampling rate) and 525 lines per frame. An interlacing procedure is used and eight bits are assigned to each sample.

Second Order Statistics: The second order statistics derived with our data base is given in table I for a 5x5 block. Note that each entry in the table represents the second order statistics $E[y_{11}y_{ij}]$ for all i and j where y_{ij} denotes an element in a 5x5 picture block. The mean value of each picture element is 143.2.

Quantizer: An optimum nonuniform quantizer (10) based on a Laplacian density function for the coefficients of the K-L transform (except for the coefficients with a nonzero mean which are not quantized) and the coefficient errors of the predictive transform coders is employed.

Bit Rate: The bit rate used for each coder is 2 bits/pel.

Bit Assignment: An optimum bit assignment was obtained for each coder using as criterion signal to noise ratio (S/N). To search for the optimum bit assignment we perturbed the bit assignment which results from assigning to each coefficient or coefficient error a number of bits which is proportional to its standard deviation (6). However, the coefficients of the K-L coders with nonzero mean were always assigned 8 bits.

Coder Matrices: In figure 4 the standard deviation and optimum bit assignments for all six coders are given. Furthermore in figure 5 the transform and predictor matrices of the 1x1 and 2x2 predictive transform coders are shown.

Simple Predictive Transform Coder:

The predictor and transform matrices $\bar{P}$ and $\bar{T}$ of the simple predictive transform 2x2 coder of figure 5c were obtained using the following procedure.

Step 1 The first column of the matrix $\bar{T}$ corresponding to the coefficient error with the largest standard deviation was selected to be

$$[1/4 \quad 1/2 \quad 1/2 \quad 3/4]^t$$

for two reasons. Firstly, these values are easily implemented and secondly these values are relatively close to those of the optimum transform matrix T of figure 5b. Note that the constant 'a' in figures 5c and 4c represents a normalizing scalar value.

Step 2 The remaining column vectors of the transform matrix T were found by searching for three vectors that satisfy the following criteria:

1. The elements of each column vector belong to the set ($\pm 1/4, \pm 1/2, \pm 3/4$).
2. The elements of each column vector have signs similar to those of the optimum elements given in figure 5b.
3. The column vectors are orthogonal and of equal energy. Note that when the matrix $\bar{T}$ is multiplied by the normalizing scalar $a = \sqrt{8/9}$ we obtain an unitary matrix T.

Step 3 An optimum predictor matrix

$$P^* = \frac{\bar{P}^*}{a} = \frac{1}{\sqrt{8/9}} \cdot \begin{bmatrix} 0.7370 & -0.2913 & -0.5070 & -0.2983 & -0.4314 & -0.9867 \\ -0.2588 & 0.1617 & 0.3070 & -0.0034 & 0.6560 & -0.4180 \\ -0.0763 & 0.2417 & -0.0353 & -0.1518 & 0.2270 & 0.2391 \\ -0.0006 & 0.0126 & 0.0510 & 0.0095 & 0.1267 & -0.1991 \end{bmatrix} \quad (25)$$

was derived by minimizing the mean square error (20) subject to the following constraints:

1. The transform matrix is unitary and fixed and given by $T=a\bar{T}$ where a and $\bar{T}$ are as defined in figure 5c.
2. The zero mean constraint (16) for the coefficient error Δc is satisfied.
3. The quantizer is assumed to work as specified by equations (18) and (19).

Step 4 The predictor matrix (25) was approximated by the matrix $\bar{P}$ given in figure 5c. It should be noted that the elements of $\bar{P}$ besides being readily implementable also approximately satisfy the zero mean constraint (16).

Step 5 The simple predictive transform 'encoder' is implemented as shown in figure 6. Note that this type of implementation avoids the need to multiply by the normalizing constant a.

Coder Performance and Complexity

In Table II the complexity of all six coders is given in the form of the number of multiplications, additions or subtractions required for the physical implementation of each coder. Note that in the case of the simple predictive transform coder the elements of the matrices $\bar{T}$ and $\bar{P}$ result in multiplications that can be easily implemented by addition or subtractions or by shifting the contents of a register.

In Table II the performance of each coder is also given using as objective criterion the signal to noise ratio (S/N)

$$S/N = 10 \log_{10}\{255^2/[\sum_{i=1}^{L^2} \sum_{j=1}^{W} (x_j(i)-\hat{x}_j(i))^2/(WL^2)]\}$$

where W, L, and $\hat{x}_j(i)$ are as defined in figure 1, and the units of S/N are in decibels (dBs). Note that the coders are organized according to their S/N ratio.

When the processed images generated by the above six coders were viewed in a standard 9 inch studio monitor the following results were obtained.

1. The K-L 2x2 coder resulted in processed images with significant blocking effects (5). The blocking effect was less severe with the K-L 4x4 coder but still noticeable. See figure 3c where the blocking effect can be seen on the boy's shoulders.
2. The optimum two-dimensional purely predictive coder, i.e., coder 3, resulted in an image with significant lack of detail (5).
3. The predictive transform coders, including the simple predictive transform coder, resulted in good quality processed images. See figure 3d where the simple predictive transform coder was used to generate the processed image.

Summary of Results:

We draw the following conclusions from our illustrative example:

1. Predictive transform coding results in readily implementable coders with high reproduction fidelity at significant compression rates. The coder that best symbolizes these properties is coder 6, i.e., the simple predictive transform 2x2 coder.
2. Predictive transform coders should be used for coder evaluation purposes rather than the classical K-L transform coders (7), (9). Note from table II that both the complexity and performance associated with the simple predictive transform 2x2 coder are superior to that of the K-L 4x4 coder.

Appendix A

In this appendix it is shown that the optimum transform and prediction matrices are found from Eqs. (21)-(23) and that the minimum MSE is given by (24). It is also shown that these optimum matrices yield uncorrelated coefficient errors.

The proof consists of the following eight steps.

Step 1: Making use of Eqs. (2), (4), (5), & (6) and the quantizer constraints (18) and (19) we have that

$$x(k+1)-\hat{x}(k+1) = \sum_{i=J+1}^{W} T_i \Delta c_i(k) \quad (A.1)$$

where J represents the number of coefficient error components unaffected by the quantizer.

Step 2: Using (A.1) in the expectation (20) we obtain

$$P_I = E[(x(k+1)-x(k+1))^t(x(k+1)-x(k+1))]$$

$$= \sum_{i=J+1}^{W} \sum_{j=J+1}^{W} T_i^t T_j E[\Delta c_i(k)\Delta c_j(k)] \quad (A.2)$$

Step 3: Using the orthonormal constraint (13) we derive

$$P_I = \sum_{i=J+1}^{W} E[\Delta c_i^2(k)] \quad (A.3)$$

Step 4: We make use of Eq. (17) to obtain

$$E[\Delta c_i^2(k)] = G_i^t E[y(k+1)y^t(k+1)] G_i \quad (A.4)$$

where

$$G_i^t = [-P_i^t T_i^t], \quad y^t(k+1) = [z^t(k) \;\; x^t(k+1)] \quad (A.5)$$

Step 5: Lagrange multipliers are used to formulate the minimization of (A.4) with respect to the vector G_i and subject to constraints (13) and (16). That is, we have the following minimization to perform

$$\min_{G_{J+1}\cdots G_W} \left\{ \sum_{i=J+1}^{W} \{G_i^t E[y(k+1)y^t(k+1)] G_i - \lambda_i (T_i^t T_i - 1) - \mu_i [U_W^t T_i - U_M^t P_i]\} \right\} \quad (A.6)$$

where λ_i and μ_i are Lagrange multipliers.

Step 6: Using standard minimization techniques [14] the minimization (A.6) is performed yielding the desired equations (21) thru (23).

Step 7: We now obtain the minimum MSE expression (24) as follows:

Firstly, $G_i^t E[y(k+1)y^t(k+1)] G_i$ is expanded to obtain

$$G_i^t E[y(k+1)y^t(k+1)] G_i$$

$$= T_i^t E[x(k+1)x^t(k+1)] T_i + P_i^t E[z(k)z^t(k)] P_i$$

$$- P_i^t E[z(k)x^t(k+1)] T_i - T_i^t E[x(k+1)z^t(k)] P_i \quad (A.7)$$

Secondly, expression (23) is solved for $E[z(k)x^t(k+1)] T_i$ to yield

$$E[z(k)x^t(k+1)] T_i = \left[E[z(k)z^t(k)] \;\; \begin{matrix} 1/2 \\ \vdots \\ 1/2 \end{matrix} \right] \begin{bmatrix} P_i \\ \mu_i \end{bmatrix} \quad (A.8)$$

Thirdly, expression (A.8) is used in (A.7) to yield

$$G_i^t E[y(k+1)y^t(k+1)] G_i = T_i^t \{E[x(k+1)x^t(k+1)] - A\} T_i \quad (A.9)$$

where A is given by (22).

Lastly, making use of the eigensystem expression (21) and the orthonormal constraint (14) in (A.9) we find that

$$G_i^t E[y(k+1)y^t(k+1)] G_i = \lambda_i \quad (A.10)$$

which implies the desired result (24).

Step 8: Next, we show that the optimum transform and predictor matrices result in uncorrelated coefficient errors.

Firstly, using an approach similar to that of steps 4 thru 7 it is shown that

$$E[\Delta c_j(k)\Delta c_i(k)] = T_j^t \{E[x(k+1)x^t(k+1)] - A\} T_i \quad (A.11)$$

where T_i, $E[x(k+1)x^t(k+1)]$, and A are as defined for (21).

Secondly, using (21) in (A.11) we find that

$$E[\Delta c_j(k)\Delta c_i(k)] = \lambda_i T_j^t T_i \text{ for all } i,j \quad (A.12)$$

Thirdly and last, using the orthonormal condition (13) in (A.12) we have the desired result

$$E[\Delta c_j(k)\Delta c_i(k)] = 0 \text{ for all } i \neq j. \quad (A.13)$$

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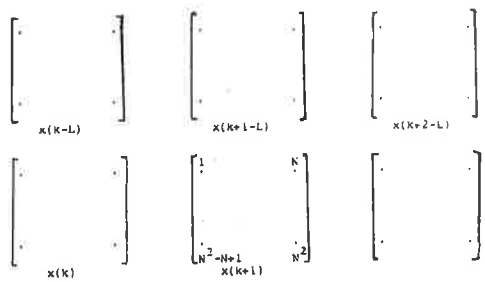


$x(i) = [x_1(i), \ldots, x_M(i)]^t$

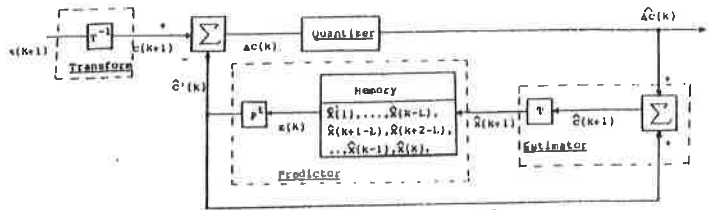
$M = N^2$

$i = 1, \ldots, L^2$

L = Number of horizontal or vertical blocks in an image field.

N = Size of each block.

Figure 1. Picture representation.



$T = [T_1, \ldots, T_M]$ where $T_i = \begin{bmatrix} t_1 \\ \vdots \\ t_M \end{bmatrix}$ $i = 1, \ldots, M$

$P = [P_1, \ldots, P_M]$ where $P_i = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}$ $i = 1, \ldots, M$

M = Number of components in the vector x(k).

Figure 2. Structure of LPT coder.

a)

$$SD = \begin{bmatrix} 253.67 & 5.20 \\ 7.21 & 2.45 \end{bmatrix} \qquad B = \begin{bmatrix} 8 & 0 \\ 0 & 0 \end{bmatrix}$$

SD = Standard Deviation Matrix B = Bit Assignment Matrix

b)

$$SD = \begin{bmatrix} 581.43 & 21.82 & 16.64 & 10.38 \\ 7.28 & 5.82 & 4.14 & 4.01 \\ 3.46 & 2.76 & 2.61 & 2.20 \\ 2.18 & 2.16 & 2.03 & 2.00 \end{bmatrix} \qquad B = \begin{bmatrix} 8 & 4 & 4 & 3 \\ 3 & 3 & 2 & 2 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

c)

$$SD = 4.01 \qquad B = 2$$

d)

$$SD = \begin{bmatrix} 9.41 & 3.70 \\ 3.00 & 2.26 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 2 \\ 2 & 0 \end{bmatrix}$$

e)

$$SD = \begin{bmatrix} 27.01 & 11.42 & 9.86 & 6.28 \\ 4.89 & 4.72 & 3.07 & 2.94 \\ 2.73 & 2.71 & 2.47 & 2.19 \\ 2.15 & 2.08 & 1.90 & 1.77 \end{bmatrix} \qquad B = \begin{bmatrix} 6 & 4 & 4 & 4 \\ 3 & 3 & 2 & 2 \\ 2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

f)

$$\tilde{SD} = \frac{1}{a} \cdot \begin{bmatrix} 9.42 & 3.75 \\ 3.05 & 2.48 \end{bmatrix} \quad a = \sqrt{8/9} \qquad B = \begin{bmatrix} 4 & 2 \\ 2 & 0 \end{bmatrix}$$

Figure 4. Standard Deviation and Bit Matrices: a) K-L Transform 2x2 case; b) K-L Transform 4x4 case; c) Predictive Transform 1x1 case; d) Predictive Transform 2x2 case; e) Predictive Transform 4x4 case; f) Simple Predictive Transform 2x2 case.

a)

$$T = 1$$

$$P^t = [0.4036 \quad -0.3270 \quad -0.2533 \quad -0.8234]$$

b)

$$T = \begin{bmatrix} 0.2764 & -0.6063 & -0.4333 & -0.6068 \\ 0.4482 & -0.6049 & 0.3876 & 0.5319 \\ 0.4362 & 0.2601 & -0.7288 & 0.4592 \\ 0.7297 & 0.4458 & 0.3618 & -0.3714 \end{bmatrix}$$

$$P^t = \begin{bmatrix} 0.7817 & -0.3138 & -0.5394 & -0.3135 & -0.4674 & -1.0361 \\ -0.2730 & 0.1908 & 0.3169 & -0.0258 & 0.7270 & -0.4197 \\ -0.0601 & 0.2202 & -0.0748 & -0.1492 & 0.1225 & 0.3542 \\ 0.0602 & 0.0625 & -0.0490 & -0.0703 & 0.0755 & -0.0920 \end{bmatrix}$$

c)

$$\tilde{T} = \begin{bmatrix} 1/4 & -1/2 & -(1-1/4) & -1/2 \\ 1/2 & -(1-1/4) & 1/2 & 1/4 \\ 1/2 & 1/4 & -1/2 & (1-1/4) \\ (1-1/4) & 1/2 & 1/4 & -1/2 \end{bmatrix}$$

$$T_i = \tilde{T}_i \cdot a \qquad i = 1,2,3,4$$

$$a = \sqrt{8/9}$$

$$\tilde{P}^t = \begin{bmatrix} (1-1/4-1/32+1/256) & -1/4 & -1/2 & -1/2 & -1/2 & -1 \\ -1/4 & 1/8 & (1/4+1/8) & 0 & (1/2+1/8) & (-1/2+1/16+1/128) \\ -1/16 & 1/4 & -1/32 & (1/4-1/32-1/128) & 1/4 & 1/4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$P_i = \tilde{P}_i / a \qquad i = 1,2,3,4$$

where

$$U_M^t T_i - U_M^t P_i = 0$$

Figure 5. Transform and Predictor Matrices: a) Predictive Transform 1x1 case; b) Predictive Transform 2x2 case; c) Simple Predictive Transform 2x2 case.

i \ j	1	2	3	4	5
1	21193	21177	21149	21117	21085
2	21164	21154	21129	21100	21071
3	21116	21109	21091	21068	21043
4	21078	21072	21058	21040	21018
5	21044	21040	21028	21012	20994

Table I. Second Order Moments

$$c(k+1) = Tx(k+1) \qquad \hat{c}'(k) = P^t z(k)$$

$$T = a\tilde{T} \quad \text{and} \quad P = \tilde{P}/a$$

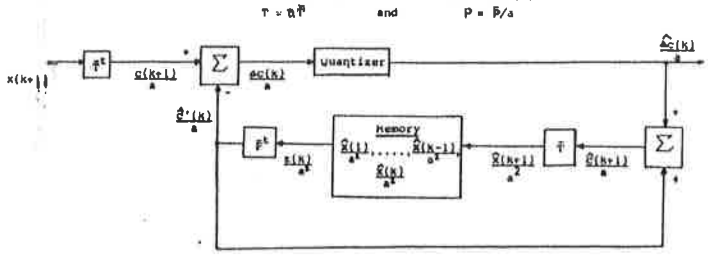
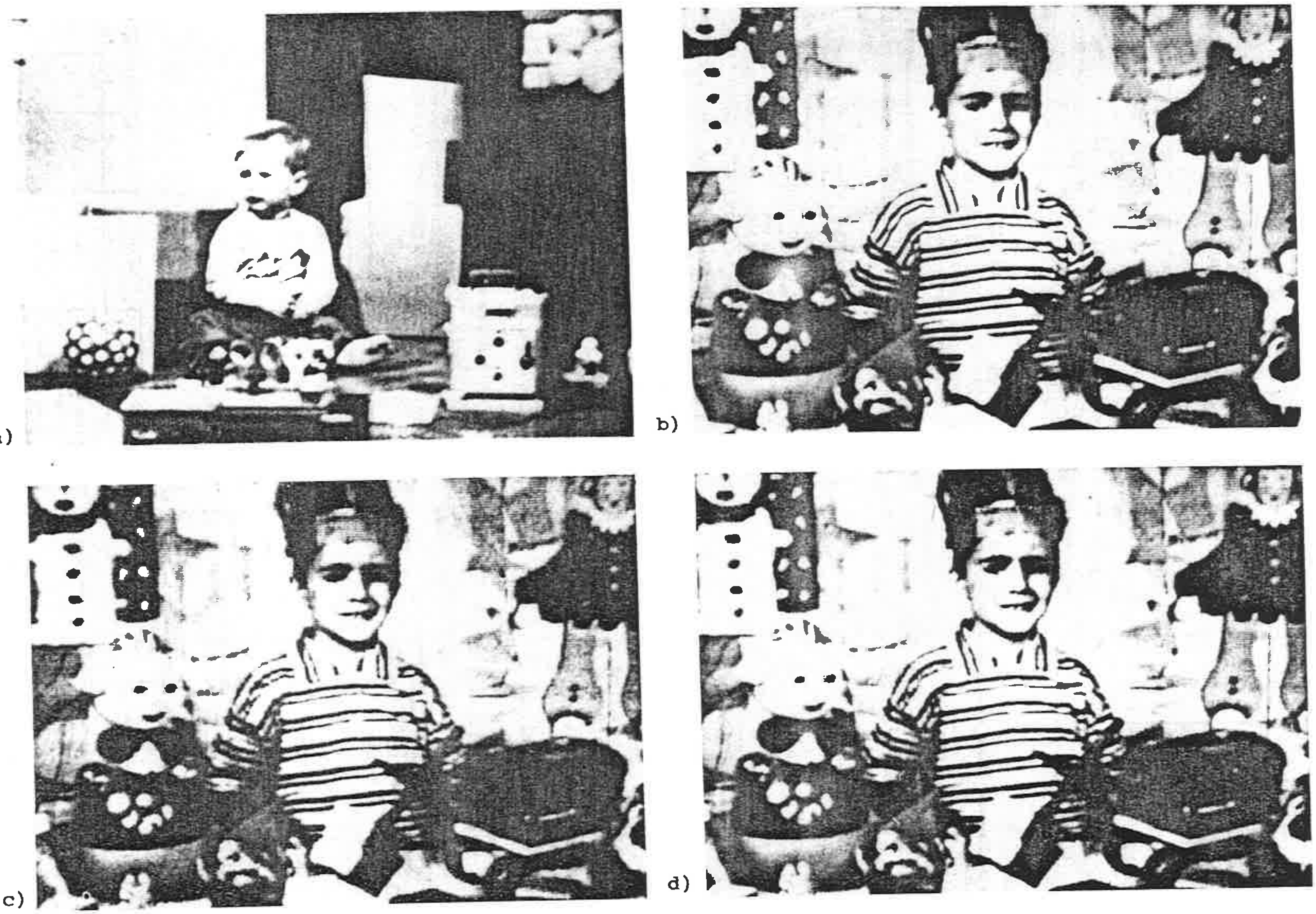


Figure 6. Simple Predictive Transform Encoder.

Coder Type	Boy with Indian Headdress S/N (dB)	Other boy S/N (dB)	Encoder Complexity	
			x	+ or -
K-L 2x2	34.03	36.28	None	8
Pred. Trans. 1x1	38.64	40.23	4	4
K-L 4x4	40.23	42.25	256	240
Simple Pred. Trans. 2x2	41.33	43.20	None	57
Pred. Trans. 2x2	41.56	43.33	40	40
Pred. Trans. 4x4	42.81	44.31	416	416

Table II



a) b) c) d)

Figure 3. a) Original Image (8 bits/sample). b) Original Image (8 bits/sample). c) K-L Transform Image (2 bits/sample). d) Simple Predictive Transform (2 bits/sample).