The Latency Information Theory Revolution, Part II: Its Statistical Physics Bridges and the Discovery of the Time Dual of Thermodynamics

Erlan H. Feria
Department of Engineering Science and Physics, City University of New York/CSI
E-mail feria@mail.csi.cuny.edu Web site http://feria.csi.cuny.edu

Abstract—Statistical physics bridges for latency information theory (LIT) are revealed in this second paper of a three paper series that include the discovery of the time dual of thermodynamics. LIT is the universal guidance theory for efficient system designs that has inherently surfaced from the confluence of five ideas. They are: 1) The source entropy and channel capacity performance bounds of Shannon's mathematical theory of communication; 2) The latency time (LT) certainty of Einstein's relativity theory; 3) The information space (IS) uncertainty of Heisenberg's quantum physics; 4) The black hole Hawking radiation and its Boltzmann thermodynamics entropy ***S*** in SI *J/K*; and 5) The author's 1978 conjecture of a structural-physical LT-certainty/IS-uncertainty duality for stochastic control. LIT is characterized by a four quadrants revolution with two mathematical-intelligence quadrants and two physical-life ones. Each quadrant of LIT is assumed to be physically independent of the others and guides its designs with an entropy if it is IS-uncertain and an ectropy if it is LT-certain. While LIT's physical-life quadrants I and III address the efficient use of life time by physical signal movers and of life space by physical signal retainers, respectively, its mathematical-intelligence quadrants II and IV address the efficient use of intelligence space by mathematical signal sources and of processing time by mathematical signal processors, respectively. Seven results are stated next that relate to the revelation of statistical physics bridges for LIT. They are: 1) Thermodynamics, a special case of statistical physics, has a time dual named lingerdynamics; 2) Lingerdynamics has a dimensionless lingerdynamics-ectropy ***Z*** that is the LT-certainty dual of a dimensionless thermodynamics-entropy, and like thermodynamics has four physical laws that drive the Universe; 3) ***S*** advances a bridge between quadrant II's source-entropy ***H*** in *bit* units and quadrant III's retainer-entropy ***N*** in SI m^2 units; 4) ***Z*** advances a bridge between quadrant I's mover-ectropy ***A*** in SI *secs* and quadrant IV's processor-ectropy ***K*** in binary operator (*bor*) units; 5) Statistical physics bridges are discovered between the LIT entropies and the LIT ectropies; 6) Half of the statistical physics bridges between the LIT entropies and LIT ectropies are found to be medium independent, thus yielding the same entropy-ectropy relationships for black holes, ideal gases, biological systems, etc.; and 7) A medium independent quadratic relationship $\tau = l(M/\Delta M)^2$ relates the lifespan τ of a retained mass M to the ratio of M to the fractional mass ΔM that escapes it every l seconds, e.g., for a human with M = 70 *kg*, expected lifespan of τ=83.9 years (or 2.65 *Gsec*), l=1 day (or 86.4 *ksec*), its daily escaping mass is given by ΔM=0.4 *kg*. In turn, this requires him/her to consume 2,000 *kcal per day* (i.e., 5,000 *kcal/kg* times 0.4 *kg*) to replace the 0.4 *kg* lost from day to day which correlates well with expectations.

Index Terms—Statistical physics, information space uncertainty, latency time certainty, communication through channels, observation across sensors, adaptive radar, black holes, ideal gas, lifespan, relativity, quantum physics

1. Introduction

This paper reveals statistical physics bridges for latency information theory (LIT) that include the discovery of the latency time (LT) certainty dual of information space (IS) uncertainty thermodynamics. A review paper that presents some preliminary results of these revelations can also be found in the IEEE Sarnoff 2010 Symposium Proceedings [1]. Moreover, two related publications [2]-[3] complement this manuscript. While in [2] a review of the control roots of LIT is advanced, in [3] the LIT roots of knowledge-unaided power-centroid adaptive radar are discussed. The manuscript is organized in three additional sections. In Section 2 the eight performance bounds that LIT uses to guide system designs are defined and illustrated with simple examples to facilitate the understanding of the derived statistical physics bridges. In Section 3 the statistical physics bridges are discussed in some detail, inclusive of its enhanced version that contains lingerdynamics, which is the newly discovered LT-certainty dual of IS-uncertainty thermodynamics. In the last Section 4 a human lifespan example is used to illustrate a result that has surfaced from the investigation of the statistical physics bridges for LIT. This result is that a quadratic relationship relates the lifespan of a mass to the ratio of this mass over the

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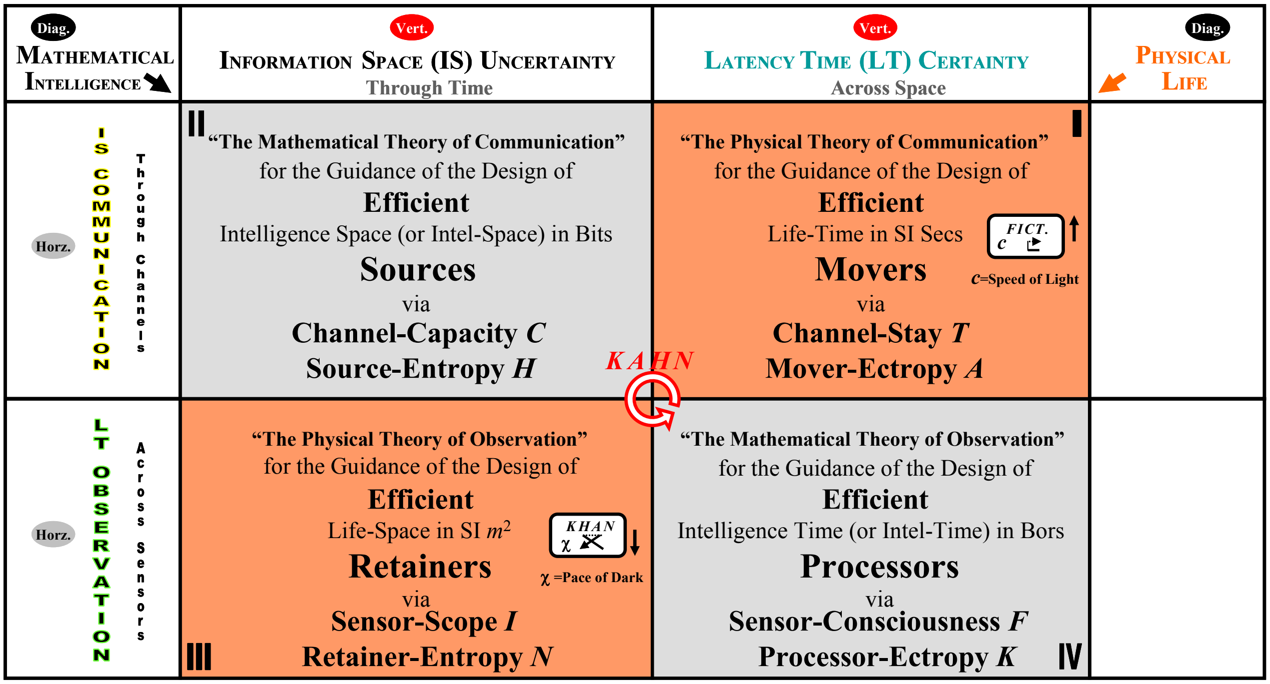

Fig. 1. The LIT revolution with four lower performance bounds [***KHAN*** (χ)], four upper performance bounds [***FICT.*** (***c***)], and the counter-clockwise ectropies to entropies statistical physics bridge sequence [$\boldsymbol{K} \Rightarrow \boldsymbol{A} \Rightarrow \boldsymbol{H} \Rightarrow \boldsymbol{N}$].

fractional mass that escapes it over some specified cyclic time span, e.g. the 86,400 seconds of a single day.

2. The Performance Bounds of the LIT Revolution

Fig. 1 displays the four quadrants of the LIT revolution inclusive of the eight performance bounds of the mathematical-physical theory of communication-observation which is part of LIT [1]. These four quadrants address physically independent system design efficiency problems whose performance bounds are nevertheless bridged by statistical physics bridges as is found in this paper. Following a counter-clockwise description these LIT quadrants are: 1) The mathematical-intelligence LT-certainty/LT-observation quadrant IV that via lower/upper performance bounds guides the design of processors with an efficient intelligence time (or intel-time), thus giving rise to what is called here, "the mathematical theory of observation". The efficiency of the processors is measured by the maximum number of binary operator (or bor) levels that the mathematical signal (or intelligence) uses as it is processed via multiple paths from start to finish. The lower bound is the processor-ectropy ***K*** in mathematical bor units that guides the design of efficient intel-time signal-processors. The other upper bound is the dimensionless sensor-consciousness ***F*** that guides the design of efficient sensor and processor integrated (SPI) coders to be described later; 2) The physical-life LT-certainty/IS-communication quadrant I that via lower/upper performance bounds guides the design of movers with an efficient life-time, thus giving rise to what is called here, "the physical theory of communication". The efficiency of the movers is measured by the maximum number of SI seconds that the physical signal uses as it is moved via multiple paths from start to finish. The lower bound is the mover-ectropy ***A*** in physical SI sec units that guides the design of efficient life-time signal-movers. The other upper bound is the dimensionless channel-stay ***T*** that guides the design of efficient channel and mover integrated (CMI) coders; 3) The mathematical-intelligence IS-uncertainty/IS-communication quadrant II that via lower/upper performance bounds guides the design of sources with an efficient intelligence space (or intel-space), thus giving rise to what Claude E. Shannon called, "the mathematical theory of communication". The efficiency of the sources is measured by the expected amount of inter-space that the mathematical signal uses as it is sourced from start to finish. The lower bound is the source-entropy ***H*** in mathematical binary digit (bit) units that guides the design of efficient intel-space signal-sources. The other upper bound is the dimensionless source-capacity ***C*** that guides the design of efficient channel and source integrated (CSI) coders; and 4) The physical-life IS-uncertainty/LT-observation quadrant III that via lower/upper performance bounds guides the design of retainers with

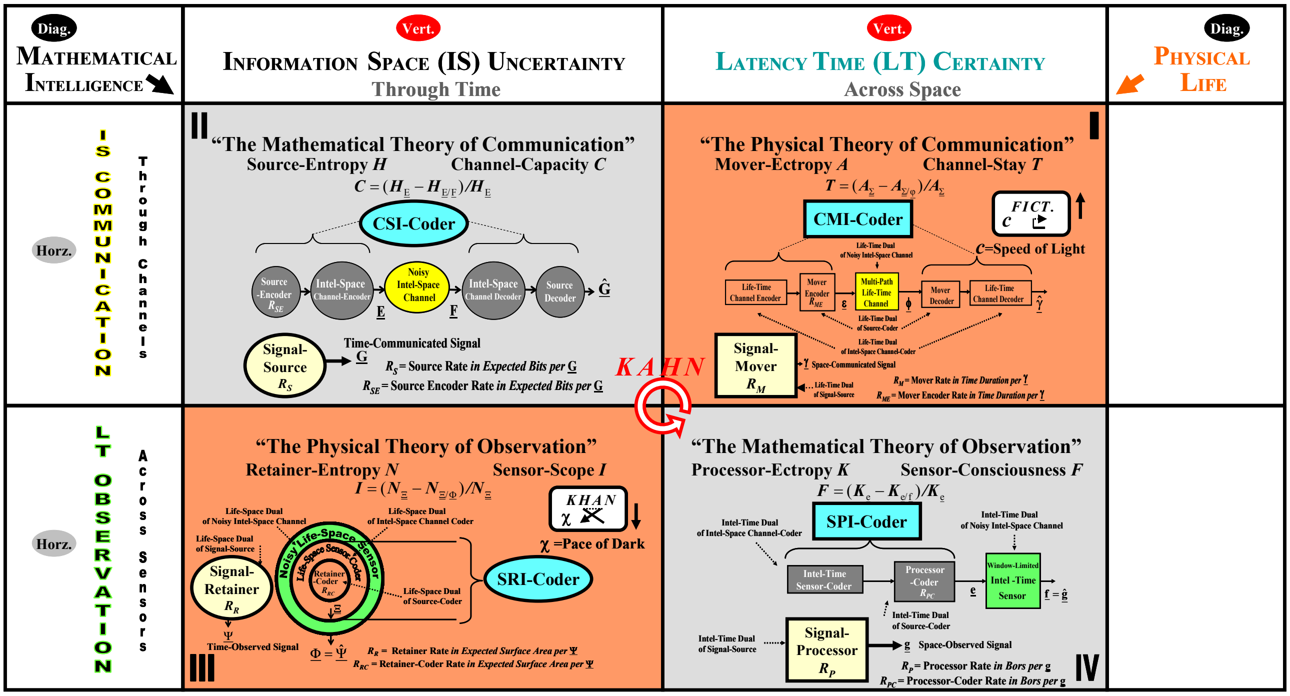

Fig. 2. The Structural-Physical LT-Certainty/IS-Uncertainty Dualities of the LIT Revolution.

an efficient life-space, thus giving rise to what is called here, "the physical theory of observation". The efficiency of the retainer is measured by the expected amount of life-space that the physical signal uses as it is retained from start to finish. The lower bound is the retainer-entropy ***N*** in physical surface area SI m^2 units that guides the design of efficient life-space signal-retainers. The other upper bound is the dimensionless sensor-scope ***'I'*** that guides the design of efficient sensor and retainer integrated (SRI) coders.

The LIT revolution exhibits three major dualities. They are: 1) The vertical LT-certainty/IS-uncertainty duality of quadrants I, IV and quadrants II, III; 2) The horizontal IS-communication/LT-observation duality of quadrants I, II and quadrants III, IV, where there are IS-uncertainty and LT-certainty versions for both channels and sensors; and c) The diagonal physical-life/mathematical-intelligence duality of quadrants I, III and quadrants II, IV, which are the two fundamental and complementary pillars of biological systems. More specifically, one of these pillars is responsible for the storage and processing of intelligence (the neural networks) and the other for the motion and retention of life which is enabled by the stored and processed intelligence. Thus from this LIT revolution a nascent efficiency theory for both living and non-living systems inherently emerges. An obvious question that then surfaces is, "Is there a natural bridge that may be used to navigate the LIT quadrants?". As will be seen in Section 3 the answer to this question is on the affirmative. The desired bridge is advanced by statistical physics in both its classical IS-uncertainty thermodynamics form [4]-[5] as well as a newly discovered LT-certainty lingerdynamics duality form.

A. The Two Performance Bounds of "The Mathematical Theory of Communication" of LIT's Quadrant II

The source-entropy ***H*** in bit units is the first performance bound. It is the expected source-information given by

$$\boldsymbol{H} = E[I_S(\underline{g}_i)] = \sum_{i=1}^{\Omega} P_S(\underline{g}_i) I_S(\underline{g}_i) = \log_2 \Lambda \tag{1}$$

$$I_S(\underline{g}_i) = \log_2(1/P_S(\underline{g}_i)) \tag{2}$$

$$\Lambda = 2^{\sum_{i=1}^{\Omega} P_S(\underline{g}_i) I_S(\underline{g}_i)} \tag{3}$$

where: 1) $\underline{G} \in \{g_1, \ldots, g_\Omega\}$ is a *n*-dimensional random vector composed of Ω vector outcomes $\{g_1, \ldots, g_\Omega\}$; 2) $I_S(g_i)$ is the g_i source-information in bit units; 3) $P_S(g_i)$ is the g_i source-probability; and 4) Λ may be viewed as an *effective* number of outcomes, with $\Lambda=\Omega$ for equally likely outcomes. Expression (1) advances a lower performance bound for the intel-space of lossless source-coders. A source-coder is any replacement of a given signal-processor. The source-coder is lossless when its output is the same as that of the given signal-source and lossy when it is not.

The dimensionless channel-capacity $\boldsymbol{C}$ is the second performance bound [6]-[7]. It is the maximum percentage of the expected source-information that can be extracted without loss from a noisy intel-space channel and is defined by

$$0 \leq \boldsymbol{C} = (\boldsymbol{H}_{\underline{\mathbf{E}}} - \boldsymbol{H}_{\underline{\mathbf{E}}/\underline{\mathbf{F}}})/\boldsymbol{H}_{\underline{\mathbf{E}}} = \max_{\{P_S[\lambda_i]\}} [(\boldsymbol{H}_{\underline{\lambda}} - \boldsymbol{H}_{\underline{\lambda}/\underline{\rho}})/\boldsymbol{H}_{\underline{\lambda}}] \leq 1 \tag{4}$$

where $\underline{\mathbf{E}}$ is the input and $\underline{\mathbf{F}}$ is the output of the channel corresponding to the n-dimensional codewords $\underline{\lambda}$ and $\underline{\rho}$ with a source-probability distribution $\{P_S[\lambda_i]\}$ that maximizes the mutual source information $(\boldsymbol{H}_{\underline{\lambda}} - \boldsymbol{H}_{\underline{\lambda}/\underline{\rho}})/\boldsymbol{H}_{\underline{\lambda}}$ (e.g., for a memoryless binary symmetric channel $\{P_S[\lambda_i]\}$ is uniformly distributed, i.e., $P_S[\lambda_1]=P_S[\lambda_2]=1/2$ [7]). In particular, $\boldsymbol{H}_{\underline{\mathrm{E}}/\underline{\mathrm{F}}}$ is a *channel-induced intel-space penalty* whose value determines the percentage of the intel-space specified by $\boldsymbol{H}_{\underline{\mathrm{E}}}$ that can be time-communicated without loss (or equivalently its probability of error approaches zero). In quadrant II of the LIT revolution of Fig. 2 the CSI-coder is displayed whose design is guided by $\boldsymbol{C}$. While the CSI-coder's source-coder efficiently compresses intel-space, its channel-coder efficiently uses overhead intel-space for the time-communication of intel-space through a noisy intel-space channel.

B. The Two Performance Bounds of "The Mathematical Theory of Observation" of LIT's Quadrant IV

The processor-ectropy $\boldsymbol{K}$ in bor units is the first performance bound. It is the minimax processor-latency given by

$$\boldsymbol{K} = \max[L_P(g_1),..,L_P(g_n)] = \max[f_1[C_P(g_1)],..,f_n[C_P(g_n)]] \tag{5}$$

where: 1) $\underline{g}=[g_1,..g_n]$ is the n-dimensional signal-processor vector output; 2) $L_P(g_i)$ is the g_i processor-latency; and 3) the function $f_i[C_P(g_i)]=L_P(g_i)$ conveys the dependence of $L_P(g_i)$ on the g_i processor-constraint $C_P(g_i)$. Expression (5) provides a lower performance bound for the intel-time of lossless processor-coders. A processor-coder is any replacement of a given signal-processor. The processor-coder is lossless when its output is the same as that of the given signal-processor and lossy when it is not. As an illustration of the use of (5) in guiding the design of either lossless or lossy processor-coders consider a 1-bit full-adder [8] signal-processor that has a slow bor multi-level implementation structure where the sum output is associated with six bor levels and the carry-out with five bor levels. This signal-processor is thus characterized by a minimax processor rate R_{P}=6 bors which is the maximum of the six bor levels for the sum, and the five bor levels for the carry-out. The reason for this relatively large number of bor levels is that this full-adder was originally designed under the implementation *processor constraints* $C_P(sum)$ and $C_P(carry\text{-}out)$ that specify that in the generation of the sum and carry-out only two-input gates can be used. Nevertheless, this same signal-processor is noted to have a processor-ectropy of 3 bors when the processor constraints are relaxed to allow for gates with more than two inputs. More specifically, from the sum of minterms Boolean expressions for the sum and carry-out of the 1-bit full-adder [8] it follows that the processor-latencies are given for the sum output by $L_P(sum)$=3 bors and for the carry-out by $L_P(carry\text{-}out)$=2 bors. The maximum of these two numbers is then the processor-ectropy $\boldsymbol{K}$=3 bors. Moreover, while the 1-bit full adder is a lossless processor-coder, a lossy but faster, by one bor level, 1-bit full adder can be readily derived from the lossless case by only implementing the two bor levels for the carry-out and by setting the sum output to zero. Thus this lossy processor-coder has a rate R_{PC} of two bors which is less than the processor-ectropy of 3 bors, i.e., $R_{\mathrm{PC}} = 2$ bors < $\boldsymbol{K}$=3 bors.

The dimensionless sensor-consciousness $\boldsymbol{F}$ is the second performance bound. It is the maximum percentage of the minimax processor-latency that can be extracted without loss by a window-limited intel-time sensor and is defined by

$$0 \leq \boldsymbol{F} = (\boldsymbol{K}_{\underline{\mathbf{e}}} - \boldsymbol{K}_{\underline{\mathbf{e}}/\underline{\mathbf{f}}})/\boldsymbol{K}_{\underline{\mathbf{e}}} = \max_{\{C_P[e_i]\}} [(\boldsymbol{K}_{\underline{e}} - \boldsymbol{K}_{\underline{e}/\underline{f}})/\boldsymbol{K}_{\underline{e}}] \leq 1 \tag{6}$$

where $\underline{\mathbf{e}}$ is the input and $\underline{\mathbf{f}}$ is the output of the sensor corresponding to the n-dimensional vectors $\underline{e}$ and $\underline{f}$ with processor-constraints $\{C_P[e_i]\}$ that maximize the mutual processor latency $(\boldsymbol{K}_{\underline{e}} - \boldsymbol{K}_{\underline{e}/\underline{f}})/\boldsymbol{K}_{\underline{e}}$ (e.g., for the full adder case the processor constraints $\{C_P[e_i]\}$ that maximize the mutual processor latency is when the sum output and carry-out can be derived using logic gates with an arbitrary number of inputs). In particular, $\boldsymbol{K}_{\underline{\mathrm{e}}/\underline{\mathrm{f}}}$ is a *sensor-induced intel-time penalty* whose value determines the percentage of the intel-time specified by $\boldsymbol{K}_{\underline{\mathrm{e}}}$ that can be space-observed without loss. In quadrant IV of the LIT revolution of Fig. 2 the SPI-coder is displayed whose design is guided by $\boldsymbol{F}$. While the SPI-coder's processor-coder efficiently compresses intel-time, its sensor-coder efficiently uses overhead intel-time for the space-observation of intel-time across a window-limited intel-time sensor. As an illustration of how (6) can be used consider a 1-bit full-adder based recursive adder of two bytes. This recursive adder has a processor-ectropy of 16 bors, i.e., $\boldsymbol{K}_{\underline{\mathrm{e}}}$=16 bors, since the processor-latency of the 1-bit full-adder carry-out is of 2 bors and 8 bit pairs (plus the carry-in for each pair) are being added. Then if one observes the adder output with a 14-bors window-limited intel-time sensor, the sensor-induced inter-time penalty will be of 2 bor, i.e. $\boldsymbol{K}_{\underline{\mathrm{e}}/\underline{\mathrm{f}}}$ =2 bors. In turn, this results in a sensor-consciousness value of $\boldsymbol{F}$=(16-2)/16=0.88 that informs us that only 88% of the 16 bors intel-time of $\boldsymbol{K}_{\underline{\varepsilon}}$ can be space-observed without loss. Thus the adder intel-time latency must be of at least 18 bors. The additional 2 bors that are required to observe the full sum can then be facilitated by a sensor-coder that uses prior-knowledge, e.g. that LSBs can be zero, which allows the addition to start 2 bors earlier in time.

C. The Two Performance Bounds of "The Physical Theory of Communication" of LIT's Quadrant I

The mover-ectropy $\boldsymbol{A}$ in SI sec units is the first performance bound. It is the minimax mover-latency given by

$$\boldsymbol{A} = \max[L_M(\gamma_1),..,L_M(\gamma_n)] = \max[f_1[C_M(\gamma_1)],..,f_n[C_M(\gamma_n)]] \tag{7}$$

where: 1) $\underline{\gamma}$=[$\gamma_1,..\gamma_n$] is the n-dimensional signal-mover vector output; 2) $L_M(\gamma_i)$ is the γ_i mover-latency; and 3) the function $f_i[C_M(\gamma_i)]=L_M(\gamma_i)$ conveys the dependence of $L_M(\gamma_i)$ on the γ_i mover-constraint $C_M(\gamma_i)$. Similarly to the processor-ectropy $\boldsymbol{K}$ (5) the mover-ectropy $\boldsymbol{A}$ (7) is a minimax criterion that advances a lower performance bound for the life-time of lossless mover-coders. A mover-coder is any replacement of a given signal-mover. The mover-coder is lossless when it moves all the given signal-mover physical signals and lossy when it does not. Examples of mover-coders are four-wheeled vehicles whose goal is the negligible decrease of the *lingering* (or remaining life-time) of people when space-dislocating them by some desired Δx_{Pe}, and photons that carry electromagnetic radiation at the speed of light in a vacuum for some desired space dislocation Δx_{Ph}. An example of a lossy mover-coder is an automobile that can only move six people, but yet replaces a van that carries ten people, thus the four people left behind represent a physical signal loss.

The mover-ectropy is next derived for a sphere that acts as a signal computational medium in a multi-path environment (in the next subsection the mover-ectropy of this sphere will be related to its retainer-entropy $\boldsymbol{N}$ when it also acts as a signal storage medium). First it is noted that the minimax property of the mover-ectropy is inherent when several movers using different computational paths simultaneously depart from the same location in space to another location in space some distance away. The paths that these movers can follow are part of a set of motion constraints, e.g., all the computational paths that can be taken along the surface of a sphere or within. For the considered spherical case the movers will be assumed to move at a constant speed v along all the computational paths available. $\boldsymbol{A}$ will then be given by

$$\boldsymbol{A}=\pi r/v \tag{8}$$

where r is the radius of the sphere. To derive this result it is first noted that $\pi r/v$ is the minimum life-time for motions that are restricted to the surface of the sphere. On the other hand, $2r/v$ is the minimum life-time for motions that are not restricted as to which path may be taken. Notice that this minimum life-time path is along the diameter of the sphere whose distance is $2r$. The largest of these two life-times, i.e. $\pi r/v$, is then the minimax mover-ectropy $\boldsymbol{A}$ of the spherical computational medium (8).

The dimensionless channel-stay T is the second performance bound. It is the maximum percentage of the expected mover-latency that can be extracted without loss from a multi-path life-time channel and is defined by

$$0 \le \boldsymbol{T} = (\boldsymbol{A}_{\underline{\varepsilon}} - \boldsymbol{A}_{\underline{\varepsilon}/\underline{\varphi}})/\boldsymbol{A}_{\underline{\varepsilon}} = \max_{\{C_M[e_i]\}} [(\boldsymbol{A}_{\underline{\varepsilon}} - \boldsymbol{A}_{\underline{\varepsilon}/\underline{\phi}})/\boldsymbol{A}_{\underline{\varepsilon}}] \le 1 \tag{9}$$

where $\underline{\varepsilon}$ is the input and $\underline{\boldsymbol{\varphi}}$ is the output of the channel corresponding to the n-dimensional vectors $\underline{\varepsilon}$ and $\underline{\phi}$ with mover-constraints $\{C_M[e_i]\}$ that maximize the mutual mover latency $(\boldsymbol{A}_{\underline{\varepsilon}} - \boldsymbol{A}_{\underline{\varepsilon}/\underline{\phi}})/\boldsymbol{A}_{\underline{\varepsilon}}$. In particular, $\boldsymbol{A}_{\underline{\varepsilon}/\underline{\varphi}}$ is a *channel-induced life-time penalty* whose value determines the percentage of the life-time specified by $\boldsymbol{A}_{\underline{\varepsilon}}$ that can be space-communicated without loss. In quadrant I of the LIT revolution of Fig. 2 the CMI-coder is displayed whose design is guided by $\boldsymbol{T}$. The c also shown in quadrant I reminds us of the Einstein conjecture of the 'speed of light in a vacuum' upper limit that movers can never exceed. While the CMI-coder's mover-coder efficiently compresses life-time, its channel-coder efficiently uses overhead life-time for the space-communication of life-time through a multi-path life-time channel. For instance, if for our spherical computational medium example 1.2 msec is derived for its minimum surface path and 1 msec for its direct diameter path, then $\boldsymbol{A}_{\underline{\varepsilon}}$ will be equal to 1.2 msec. Thus if the movement along each motion path is slowed down by a life-time channel that increases each mover life-time by at most 0.2 msec, it then follows that $\boldsymbol{A}_{\underline{\varepsilon}/\underline{\phi}}$ will be of 0.2 msec. In turn, this results in $\boldsymbol{T}$=(1.2-0.2)/1.2=0.834 which informs us that only 83.3% of the 1.2 msec life-time in $\boldsymbol{A}_{\underline{\varepsilon}}$ can be space-communicated without loss. Thus the life-time of the longest life-time mover can never be less than 1.4 msecs. It is then the task of the life-time channel coder to provide the mover paths that satisfy the 1.4 msecs limit.

D. The Two Performance Bounds of "The Physical Theory of Observation" of LIT's Quadrant III

The retainer-entropy $\boldsymbol{N}$ in SI m^2 units is the first performance bound. It is the expected retainer-information given by

$$\boldsymbol{N} = E[I_R(\underline{\varepsilon}_i)] = \sum_{i=1}^{\Omega} I_R(\underline{\varepsilon}_i) P_R(\underline{\varepsilon}_i) = 4\pi r^2 \quad (10)$$

$$I_R(\underline{\varepsilon}_i)=4\pi r_i^2(P_R(\underline{\varepsilon}_i)) \quad (11)$$

$$r = \sqrt{\sum_{i=1}^{\Omega} r_i^2(P_R(\underline{\varepsilon}_i)) P_R(\underline{\varepsilon}_i)} \quad (12)$$

where:1) $\underline{\Psi} \in \{\underline{\varepsilon}_{1,\dots,}\underline{\varepsilon}_{\Omega}\}$ is a n-dimensional random vector composed of Ω vector outcomes (or microstates) $\{\underline{\varepsilon}_{1,\dots,}\underline{\varepsilon}_{\Omega}\}$; 2) $I_R(\underline{\varepsilon}_i)$ is the $\underline{\varepsilon}_i$ retainer-information in SI m^2, which specifies the minimum surface area (corresponding to the surface area of a sphere of radius $r_i(.)$ and volume V_i) life-space of $\underline{\varepsilon}_i$ with volume V_i; 3) $P_R(\underline{\varepsilon}_i)$ is the $\underline{\varepsilon}_i$ retainer-probability; and 4) r is the radius of the sphere given by the square root of the expected radius square of the minimum surface area spheres linked to $\{\underline{\varepsilon}_{1,\dots,}\underline{\varepsilon}_{\Omega}\}$. Similarly to the source-entropy $\boldsymbol{H}$ (1) the retainer-entropy $\boldsymbol{N}$ (10) is an expectation criterion that advances a lower performance bound for the life-space of lossless retainer-coders. A retainer-coder is any replacement of a given signal-retainer. The retainer-coder is lossless when it retains all the given signal-retainer physical signals and lossy when it does not. Examples of retainer-coders are a thermos whose goal is the negligible decrease of the *temperature* of hot tea when time-dislocating it by some desired $\Delta\tau_{Ht}$, and an atom that maintains the direction of its spin for some desired time-dislocation $\Delta\tau_{Sp}$. An example of a lossy retainer-coder is a thermos that can only store three hot tea servings, but yet replaces a thermos that stores five hot tea servings, thus the two hot tea servings left behind represent a physical signal loss. An example of a lossless retainer-coder that achieves the retainer-entropy $\boldsymbol{N}=4\pi r^2$ is a spherical thermos of hot tea whose volume is the same as that of the given cylindrical thermos that it replaces.

The dimensionless sensor-scope '$\boldsymbol{I}$' is the second performance bound. It is the maximum percentage of the expected retainer-information that can be extracted without loss from a noisy life-space sensor and is defined by

$$0 \le \boldsymbol{I} = (\boldsymbol{N}_{\underline{\Xi}} - \boldsymbol{N}_{\underline{\Xi}/\underline{\Phi}})/\boldsymbol{N}_{\underline{\Xi}} = \max_{\{P_R[\underline{\beta}_i]\}} [(\boldsymbol{N}_{\underline{\beta}} - \boldsymbol{N}_{\underline{\beta}/\underline{\alpha}})/\boldsymbol{N}_{\underline{\beta}}] \le 1 \quad (13)$$

where $\underline{\Xi}$ is the input and $\underline{\Phi}$ is the output of the sensor corresponding to the n-dimensional microstates $\underline{\beta}$ and $\underline{\alpha}$ with a retainer-probability distribution $\{P_R[\underline{\beta}_i]\}$ that maximizes the mutual retainer information $(\boldsymbol{N}_{\underline{\beta}} - \boldsymbol{N}_{\underline{\beta}/\underline{\alpha}})/\boldsymbol{N}_{\underline{\beta}}$. In particular, $\boldsymbol{N}_{\underline{\Xi}/\underline{\Phi}}$ is a *sensor-induced life-space penalty* whose value determines the percentage of the life-space specified by $\boldsymbol{N}_{\underline{\Xi}}$ that can be time-observed without loss. In quadrant III of the LIT revolution of Fig. 2 the SRI-coder is displayed whose design is guided by '$\boldsymbol{I}$'. The $\boldsymbol{\chi}$ also shown in quadrant III reminds us about the conjecture of 2008 by the author [9] of the 'pace of dark in an uncharged and non-rotating black hole (UNBH)' upper limit that retainers can never exceed. The derived expression and value for the pace of dark is

$$\boldsymbol{\chi} = \tau/V = 960\pi c^2/hG = 6.1123 \times 10^{63} \text{ secs}/m^3 \quad (14)$$

where τ is the lifespan of a UNBH with an initial volume of V, and h and G are the Plank and gravitational constants, respectively. While the SRI-coder's retainer-coder efficiently compresses life-space, its sensor-coder efficiently uses overhead life-space for the time-observation of life-space across a noisy life-space sensor. As an illustration, if a cylindrical thermos for hot tea with a surface area of $168\pi\ cm^2$ has a retainer-entropy of $\boldsymbol{N}_{\underline{\Xi}}=144\pi\ cm^2$, this retainer-entropy can be implemented with a spherical thermos with a 6 cm radius that has the same volume as the given cylindrical thermos. However, if the hot tea is time-observed with a noisy life-space sensor consisting of random people that require the drinking of the hot tea from a thermos cup with a $166\pi\ cm^2$ surface space, the sensor-induced life-space penalty will be of $22\pi\ cm^2$, i.e. $\boldsymbol{N}_{\underline{\Xi}/\underline{\Phi}}=22\pi\ cm^2$. In turn, this results in $\boldsymbol{I}$=(144-22)/144=0.847 informing us that only 84.7% of the $144\pi\ cm^2$ life-space of $\boldsymbol{N}_{\underline{\Xi}}$ can be time-observed without loss. Thus the hot tea life-space must be of at least $166\pi\ cm^2$. It is then the task of the life-space sensor coder to provide a thermos cup that satisfies the $166\pi\ cm^2$ limit.

3. The Statistical Physics Bridges of the LIT Revolution

In this section it is revealed that statistical physics, of which thermodynamics is a special case, offers a natural bridge for the entropies and ectropies of the LIT revolution. The discussion begins with the thermodynamic-entropy for a black hole [4]-[5] that advances a natural 'linear bridge' between the source-entropy and retainer-entropy of the IS-uncertainty quadrants II and III of LIT. Then for an ideal gas a 'nonlinear *logarithmic* bridge' between these two entropies is found. Following with this investigation it is then discovered that a similar type of bridge exists between the mover-ectropy of quadrant I and the retainer-entropy of quadrant III, which in turn leads to the realization that thermodynamics has a LT-certainty dual which has been called lingerdynamics [13]. As expected from this LT-certainty/IS-uncertainty duality perspective a natural bridge between the mover-ectropy of quadrant I and the processor-ectropy of quadrant IV is then revealed for both the UNBH and an ideal gas. The section then ends with a summary of the lingerdynamics terms that are the LT-certainty duals of thermodynamic terms as well as the relations that bridge them.

A. The Black Hole Thermodynamics Entropy

It is well known [4] that a linear relationship exists between the Boltzmann thermodynamics-entropy $\boldsymbol{S}$ and the Shannon source-entropy $\boldsymbol{H}$ that is given by

$$\boldsymbol{S} = \ln 2\, k\boldsymbol{H} \tag{15}$$

where both $\boldsymbol{S}$ and the Boltzmann constant k are in SI joules per kelvin (J/K) units and $\boldsymbol{H}$ is in bit units. Moreover, it is found that when the microstates of the retained mass or energy are equally likely, $\boldsymbol{H}$ attains the maximum value of $\boldsymbol{H}=\log_2\Omega$ bits and $\boldsymbol{S}$ attains the maximum value of $\boldsymbol{S}=k\ln\Omega$ as expected. For the special case of a UNBH the thermodynamic-entropy has been studied by Hawking and others [4]-[5], [9]. The principal result of this investigation that has a direct impact on the revelation of a statistical physics bridge for LIT is summarized by the relationship

$$\boldsymbol{S_{EH}} / \ln 2\, k = \pi c^3 A / 2\ln 2\, hG = \chi\, cA/1920\ln 2 = \boldsymbol{H_{EH}} = \phi_{S_{EH}} \boldsymbol{N_{EH}} = \boldsymbol{N_{EH}} / \boldsymbol{N_{Bit}} = \tau_{EH} / l_{Bit} = \left(\boldsymbol{M_{EH}} / \boldsymbol{M_{Bit}}\right)^2 \tag{16}$$

where:

1) $\boldsymbol{A}$ is the surface area of the spherical UNBH and c, h, G and $\boldsymbol{\chi}$ are the four Universe constants (14) [9].

2) The subscript $\boldsymbol{EH}$ that appears in (16) for different variables signify the 'event horizon' where a black-hole meets a vacuum. Hawking conjectured in the mid 1970's that on this event horizon [5] photon pairs are spontaneously created, with one photon in each pair emerging inside the vacuum (the so-called Hawking radiation) and the other emerging inside the black-hole. While the photon inside the vacuum increases the positive energy of the vacuum, the photon inside the black-hole decreases the positive energy of the black-hole. Thus the Hawking conjecture predicts a finite life-time for any black hole in the absence of any external mass or energy entering it. If the initial volume V_{EH} of the UNBH is known (or equivalently its initial mass M_{EH} since $V_{EH}=4\pi r^3/3=4\pi(2GM_{EH}/c^2)^3/3$ where $r = 2GM_{EH}/c^2$ is the Schwarzschild radius [4]-[5]), the lifespan τ_{BH} of the UNBH can be easily derived by multiplying the pace of dark $\boldsymbol{\chi}$ by V_{EH}, i.e.,

$$\tau_{EH} = V_{EH}\boldsymbol{\chi}. \tag{17}$$

3) $\boldsymbol{S_{EH}}$, $\boldsymbol{H_{EH}}$ and $\boldsymbol{N_{EH}}$ are the thermodynamics-entropy, source-entropy and retainer-entropy of the UNBH, respectively, with

$$\boldsymbol{N_{EH}} = A = 4\pi r^2 = 4\pi\left(2GM_{EH}/c^2\right)^2 \tag{18}$$

where the speed of light c appearing in (18) may be interpreted as the escape speed of the Hawking radiation from the event horizon of the UNBH.

4) $\boldsymbol{N_{Bit}}$ is the bit retainer-entropy derived from the expressions

$$\boldsymbol{N_{Bit}} = 1/\phi_{S_{EH}} = 2\ln 2 hG/\pi c^3 = 4\pi r_{Bit}^2 = 4\pi(2GM_{Bit}/c^2)^2 \tag{19}$$

$$\pi r_{Bit} = \sqrt{\pi \ln 2}\; L_P = 1.4757\, L_P \tag{20}$$

$$L_P = \sqrt{hG/2\pi c^3} \tag{21}$$

$$M_{Bit} = \sqrt{2\ln 2} M_{RP} = 1.1774 M_{RP} \tag{22}$$

$$M_{RP} = \sqrt{hc/16\pi^2 G} \tag{23}$$

where πr_{Bit} (denoting ½ of the circumference of the retainer-entropy sphere of radius r_{Bit}) is larger than the Plank length L_P (21) as noted from (20) and expected by theory [4]. Moreover, it is assumed that a bit has a mass M_{Bit} (22) (or energy for photons) whose escape speed approaches c (or is c for photons) exceeding the reduced Plank mass M_{RP} (23). Finally $\phi_{S_{EH}}$ is the bit retention (or storage) surface fix in SI m^{-2} units of the UNBH where surface fix is the retention dual of frequency. In Table 1 selected retention/motion dualities are stated inclusive of the aforementioned surface-fix/frequency duality.

5) l_{Bit} is the time span (or *weavelength* which is the retention dual of wavelength) of escape of the bit mass M_{Bit} from the UNBH mass M_{EH} via Hawking radiation, and is related to the UNBH radius r and the speed of light c as follows

$$l_{Bit} = \boldsymbol{N_{Bit}}\chi\, r/3 = 640\ln 2\, r/c\,. \tag{24}$$

B. The Ideal Gas Thermodynamics Entropy

From (16) it is noted that statistical physics advances an inherent link between the source-entropy $\boldsymbol{H_{EH}}$ of LIT's quadrant II and the retainer-entropy $\boldsymbol{N_{EH}}$ of LIT's quadrant III. However, the simple linear relation $\boldsymbol{H_{EH}}=\boldsymbol{N_{EH}}/\boldsymbol{N_{Bit}}$ is unique to the UNBH medium and thus must be found for other mediums. For instance, for an ideal gas (IG) [10]-[11] the following nonlinear bridge between its retainer-entropy $\boldsymbol{N_{IG}}$ and its source-entropy $\boldsymbol{H_{IG}}$ is derived in Appendix A

$$\boldsymbol{S_{IG}}/\ln 2k = \boldsymbol{H_{IG}} = J\left(\ln(\boldsymbol{V_{IG}}T^{c_V}/J\mathrm{B}) + c_P\right)/\ln 2 = J\log_2\left(\boldsymbol{N_{IG}}/\Delta\boldsymbol{N_{IG}} = \tau_{IG}/l_{IG} = \left(\boldsymbol{M_{IG}}/\Delta\boldsymbol{M_{IG}}\right)^2\right) \quad (25)$$

$$\boldsymbol{N_{IG}} = 3V_{IG}/r = 4\pi r^2 = 4\pi\left(2GM_{IG}/v_e^2\right)^2 \quad (26)$$

$$\Delta\boldsymbol{N_{IG}} = 1/\phi_{S_{IG}} = 4\pi\Delta r^2 = 4\pi\left(2G\Delta M_{IG}/v_e^2\right)^2 \quad (27)$$

$$\phi_{S_{IG}} = \sigma\left(e\,m\right)^{5/2}\left(2\pi\,kT/h^2\right)^{3/2}\left(r/M_{IG}\right)/3 \quad (28)$$

$$\sigma = g\,e^{c_P-5/2}T^{c_V-3/2} \quad (29)$$

$$m = M_{IG}/J = 3kT/v_{rms}^2 \quad (30)$$

$$r/M_{IG} = 2G/v_e^2 \quad (31)$$

$$\tau_{IG} = \boldsymbol{V_{IG}}\Pi \quad (32)$$

$$\boldsymbol{l_{IG}} = \Delta\boldsymbol{N_{IG}}\Pi\,r/3 \quad (33)$$

where: 1) J is the number of gas molecules (assumed in this illustrative case to be of one species but easily extended to multi-species via the Gibbs theorem [10]); 2) V_{IG}, T and M_{IG} are the volume, temperature and mass of the gas in SI m^3, K, and kg units, respectively; 3) c_V and $c_P=c_V+1$ are the dimensionless heat capacity constants under constant volume and pressure conditions, respectively, with $c_V=3/2$ and $c_P=5/2$ for a monatomic gas (the value of c_V can be found either experimentally or theoretically, from the degrees of freedom d_f of the molecules where $c_V=d_f/2$); 4) h is the Plank constant; 5) r is the radius of a sphere of volume V_{IG}; 6) v_e is the escape speed in SI m/sec units of the gas molecules from the gravitational field of the gas mass M that is assumed to be a point mass at the center of a sphere of radius r; 7) v_{rms} is the root mean square speed of the gas molecules; 8) $\mathrm{B} = T^{3/2}\mathrm{X}^3/g$ is an undetermined gas constant where $\mathrm{X} = h/\sqrt{2\pi mkT}$ is the thermal de Broglie wavelength, g=1 for a monatomic gas, and m is the mass of a single molecule; 9) $\phi_{S_{IG}}$ in weavelength (the retention dual of wavelength defined in Table 1 [9]) cycles$_l$/m^2 units is the retention (or storage) surface fix (the retention dual of frequency [9] defined in Table 1) of the gas; 10) σ is a dimensionless constant that has a value of one for a monatomic gas since $c_V=3/2$ and $g=1$ for this case; 11) $\boldsymbol{N_{IG}}$ is the retainer-entropy associated with M_{IG}; 12) $\Delta\boldsymbol{N_{IG}}$ is a small fraction of the retainer-entropy $\boldsymbol{N_{IG}}$ that is associated with the fractional mass $\Delta M_{IG} << M_{IG}$ whose molecules' escape speed is v_e; 13) Π is the pace of retention in SI sec/m^3 units of the ideal gas in the volume V_{IG}; 14) τ_{IG} is the lifespan of the ideal gas; and 15) l_{IG} is the cyclic time span (or weavelength) of escape from the ideal gas mass M_{IG} of the fractional mass ΔM_{IG}.

From (16) and (25) the general relationship between the retainer-entropy $\boldsymbol{N}$ and the source-entropy $\boldsymbol{H}$ is noted to be nonlinear, thus in general

$$\boldsymbol{S}/\ln 2k = \boldsymbol{S_k} = \boldsymbol{H} = f\left(\boldsymbol{N}\right) \quad (34)$$

where, in particular, $f(.)$ is a linear function of $\boldsymbol{N}$ for black holes (16) and is a nonlinear function of $\boldsymbol{N}$ for ideal gases (25), and the dimensionless thermodynamic-entropy expression $\boldsymbol{S}/\ln 2k$ has been assigned the symbol $\boldsymbol{S_k}$.

C. The Revelation of Lingerdynamics

When the spherical storage medium associated with the retainer-entropy $\boldsymbol{N}=4\pi r^2$ (10) has the dual role of serving as a spherical computational medium, it is then noted that its mover-ectropy $\boldsymbol{A}=\pi r/v$ (8) is inherently linked to its retainer-entropy via the sphere's radius r. Thus the following bridge relationship is revealed

$$\boldsymbol{A} = \sqrt{\pi\boldsymbol{N}/4v^2} \quad (35)$$

$$v_e \le v \le c \quad (36)$$

where the speed v is assumed greater than or equal to the escape speed v_e of ΔM_{IG} from the retained mass M_{IG} and less than or equal to the speed of light c. Moreover, while for an UNBH v_e=c since the escape speed from a black hole is that of Hawking radiation, for an ideal gas

$$v_e = \sqrt{2GM_{IG}/r} \tag{37}$$

since the escape speed v_e of ΔM_{IG} from the gravitation field of the point-mass M_{IG} is given by (37).

The bridge between the LT-certainty mover-ectropy $\boldsymbol{A}$ and the IS-uncertainty retainer-entropy $\boldsymbol{N}$ for a spherical medium expressed by (35) inherently leads to the revelation of a LT-certainty dual for the IS-uncertainty thermodynamics bridge from $\boldsymbol{N}$ to $\boldsymbol{H}$ (34). This LT-certainty duality expression is given by

$$\boldsymbol{Z} = \boldsymbol{K} = g(\boldsymbol{A}) \tag{38}$$

where: 1) $g(.)$ is some function of $\boldsymbol{A}$ that links the mover-ectropy $\boldsymbol{A}$ to the processor-ectropy $\boldsymbol{K}$**;** and 2) $\boldsymbol{Z}$ is the dimensionless lingerdynamics-ectropy dual of the dimensionless thermodynamics entropy $\boldsymbol{S_k}$ defined in (34). It should be noted from the equality $\boldsymbol{Z}$=$\boldsymbol{K}$ in (38) that although the lingerdynamics-ectropy $\boldsymbol{Z}$ is 'physically' dimensionless, it still has the same minimax computational mathematical bor units of $\boldsymbol{K}$. Also note from $\boldsymbol{S_k}$=$\boldsymbol{H}$ in (34) that although the dimensionless thermodynamics-entropy expression $\boldsymbol{S_k}$ is 'physically' dimensionless it still has the same expected storage mathematical bit units of $\boldsymbol{H}$. Furthermore, while the IS-uncertainty bridge (34) is part of thermodynamics, the LT-certainty bridge expression (38) is part of lingerdynamics which is the designated name for the LT-certainty dual of the IS-uncertainty thermodynamics. Notice that while the word *thermo* in thermodynamics relates to the IS-uncertainty properties of matter, the word *linger* in lingerdynamics relates to the LT-certainty properties of matter. Thus in essence statistical physics has been discovered to exhibit a LT-certainty/IS-uncertainty duality perspective which was first revealed in an October 2009 PSC-CUNY 41-951 research award proposal [13].

The bridge between $\boldsymbol{A}$ and $\boldsymbol{K}$ for the UNBH and the ideal gas are easily derived. For the UNBH it is given by the expressions

$$\boldsymbol{Z_{EH}} = \boldsymbol{K_{EH}} = \boldsymbol{A_{EH}}/\boldsymbol{A_{Bor}} = \sqrt{\boldsymbol{N_{EH}}/\boldsymbol{N_{Bit}}} \tag{39}$$

$$\boldsymbol{A_{EH}} = \sqrt{\pi \boldsymbol{N_{EH}}/4c^2} \tag{40}$$

$$\boldsymbol{A_{Bor}} = 1/f_{EH} = \sqrt{\pi \boldsymbol{N_{Bit}}/4c^2} \tag{41}$$

where $\boldsymbol{Z_{EH}}$, $\boldsymbol{K_{EH}}$ and $\boldsymbol{A_{EH}}$ are the lingerdynamics-ectropy, processor-ectropy and mover-ectropy of the UNBH, and $\boldsymbol{A_{Bor}}$ is the mover-ectropy of the spherical medium associated with the bit retainer-entropy $\boldsymbol{N_{Bit}}$ while $\boldsymbol{f_{EH}}$ is the bor motion (or computational) frequency of the UNBH. Furthermore, using (19) in (41) $\boldsymbol{A_{Bor}}$ can be expressed as

$$\boldsymbol{A_{Bor}} = \pi\, r_{Bit}/c = \sqrt{\pi \ln 2}\; T_{\mathrm{P}} = 1.4757 T_{\mathrm{P}} \tag{42}$$

$$T_{\mathrm{P}} = L_P/c \tag{43}$$

which is noted to be larger than the Plank time T_P as suggested by theory. Equations (16) and (39) can then be combined to yield the following bridge relationship between all four quadrants of LIT for an UNBH

$$\boldsymbol{S_{k,EH}} = \boldsymbol{H_{EH}} = \boldsymbol{N_{EH}}/\boldsymbol{N_{Bit}} = (\boldsymbol{A_{EH}}/\boldsymbol{A_{Bor}})^2 = \boldsymbol{K^2_{EH}} = \boldsymbol{Z^2_{EH}}\,. \tag{44}$$

$$\boldsymbol{A_{EH}} = \sqrt{\pi\, \boldsymbol{N_{EH}}/4c^2} \tag{45}$$

$$\boldsymbol{A_{Bor}} = 1/f_{EH} = \sqrt{\pi\, \boldsymbol{N_{Bit}}/4c^2} \tag{46}$$

For an ideal gas a universal statistical physics bridge can be derived using a similar methodology as that used to find the one for a UNBH (44). More specifically, one departs from (25) while assuming that the relationship between $\boldsymbol{A}$ and $\boldsymbol{N}$ for LIT's physical-life quadrants I and III is given by (35) and (36), i.e.,

$$\boldsymbol{A} = \sqrt{\pi\, \boldsymbol{N}/4v^2}\,, \tag{47}$$

and the relationship between $\boldsymbol{H}$ and $\boldsymbol{K}$ for LIT's mathematical-intelligence quadrants II and IV is similar to that for the UNBH (44), i.e.,

$$\boldsymbol{K} = \sqrt{\boldsymbol{H}}\,. \tag{48}$$

When (47) and (48) are used in conjunction with (25) the desired statistical physics bridge results

$$\boldsymbol{S_{k,IG}} = \boldsymbol{H_{IG}} = J\log_2\left(\boldsymbol{N_{IG}}/\Delta\boldsymbol{N_{IG}} = (\boldsymbol{A_{IG}}/\Delta\boldsymbol{A_{IG}})^2\right) = \boldsymbol{K^2_{IG}} = \boldsymbol{Z^2_{IG}} \tag{49}$$

$$\boldsymbol{A_{IG}} = \sqrt{\pi\, \boldsymbol{N_{IG}}/4v^2} \tag{50}$$

$$\Delta \boldsymbol{A}_{IG} = 1/f_{IG} = \sqrt{\pi\, \Delta \boldsymbol{N}_{IG} / 4v^2} \quad (51)$$

$$v_e = \sqrt{2GM/r} \le v \le c \quad (52)$$

where $\boldsymbol{Z}_{IG}$, $\boldsymbol{K}_{IG}$ and $\boldsymbol{A}_{IG}$ are the lingerdynamics-ectropy, processor-ectropy and mover-ectropy of an ideal gas, and $\Delta \boldsymbol{A}_{IG}$ is the mover-ectropy of the motion (or computational) sphere associated with the retainer-entropy $\Delta \boldsymbol{N}_{IG}$ while f_{IG} is the motion (or computational) frequency of the ideal gas.

D. Statistical Physics with Retention Variables

The defining expressions for the UNBH and the ideal gas bridge expressions (44) and (49) can also be expressed in terms of the physical retention duals of motion variables first advanced in [9] as well as the lingerdynamics dual for temperature whose assigned name is lingerature. When this is done the following expressions result:

$$\boldsymbol{S}_{EH} = \boldsymbol{N}_{EH}\, k / 4L_P^2 = \boldsymbol{N}_{EH}\, kc\boldsymbol{\chi}/1920 \quad (53)$$

$$\boldsymbol{N}_{Bit} = 1/\phi_{S_{EH}} = 4 \ln 2\, L_P^2 = 1920 \ln 2 / c\boldsymbol{\chi} = 7.2628 \times 10^{-70}\ m^2 \quad (54)$$

$$L_P = \sqrt{480/c\boldsymbol{\chi}} \quad (55)$$

$$\Delta \boldsymbol{N}_{IG} = 1/\phi_{S_{IG}} = 4\pi\left(2G\Delta M_{IG}/v_e^2\right)^2 = 4\pi\left(3/4\pi\boldsymbol{\chi}\right)^{2/3}\left(6\Phi\Delta O_{IG}/\Pi_e^2\right)^2 \quad (56)$$

$$\tau = 4\pi r^3 \boldsymbol{\chi}/3 \quad (57)$$

$$\alpha = \Phi O_{IG}/\tau^{4/3} = GM_{IG}\boldsymbol{\chi}/4\pi r^4 c^2 = a\boldsymbol{\chi}/4\pi r^2 c^2 \quad (58)$$

$$\Phi = \sqrt[3]{4\pi\boldsymbol{\chi}^{10}/81c^{12}}\ G = 1.8538 \times 10^{168}\ Pa.\sec^{4/3}/kg_R^2 \quad (59)$$

$$\Delta O_{IG} = \Delta M_{IG} c^2/\boldsymbol{\chi} \quad (60)$$

$$\Pi_e = \sqrt{6\Phi\, O_{IG}/\tau^{1/3}} = \sqrt{2G\, M_{IG}/r}\ \boldsymbol{\chi}/c = v_e\boldsymbol{\chi}/c \quad (61)$$

$$\phi_{S_{IG}} = \sigma\left(eo\boldsymbol{\chi}/c^2\right)^{5/2}\left(2\pi\, \ddot{L}\boldsymbol{\chi}/h^2\right)^{3/2}\sqrt[3]{3c^6/4\pi\boldsymbol{\chi}^4}\left(\tau^{1/3}/O_{IG}\right)/3 \quad (62)$$

$$o = mc^2/\boldsymbol{\chi} \quad (63)$$

$$\ddot{L} = kT\,\boldsymbol{\chi} \quad (64)$$

$$\Pi_{rms} = \sqrt{3\ddot{L}/o} = \sqrt{3kT/m}\,\boldsymbol{\chi}/c = v_{rms}\boldsymbol{\chi}/c \quad (65)$$

where: 1) Eqs. (60), (61), (63), (64) and (65) are statistical bridges from ΔM_{IG}, v_e, m, T and v_{rms} to the mater ΔO_{IG} in SI kg_R=$kg.m^5/sec^3$ (ΔO_{IG} is the retention dual of mass ΔM_{IG}), escape pace Π_e in SI sec/m^3, the molecular mater o in SI kg_R=$kg.m^5/sec^3$, lingerature $\ddot{L}$ in SI *Pa.sec*, and rms pace Π_{rms} in SI sec/m^3; 2) Eq. (57) is the bridge from the radius r of a motion-space vacuum sphere that at its motion-space center contains the space-point mass M_{IG}, to the retention τ of a retention-time UNBH sphere that at its retention-time center contains the time-point mater O_{IG} [9]; 3) α is the *escalation* of mater in SI sec/m^6 (α is the retention dual of the mass acceleration 'a'); 4) Eq. (58) is the bridge from the acceleration a of a mass at some point in motion-space p_s that is due to the gravitational-field in a vacuum of a space-point mass M_{IG} space-dislocated from p_s by r, to the escalation α of a mater at some point in retention-time p_t that is due to the gravidness-fallow in a UNBH (the retention dual of the gravitational-field in a vacuum) of a time-point mater O_{IG} time-dislocated from p_t by τ; 5) Φ is the gravidness constant; 6) Eq. (59) is the bridge from G to Φ; 7) Eqs. (56) and (62) are retention duals for (27) and (28); and 8) Eqs. (53)-(55) express $\boldsymbol{S}_{EH}$, $\boldsymbol{N}_{Bit}$ and L_P in terms of $c\boldsymbol{\chi}$. It is of interest to note that while the value of $\boldsymbol{N}_{Bit}$ given in (54) sets a lower limit for the retainer-entropy of any medium, the retention (or storage) surface fix $\phi_{S_{EH}}$ of the UNBH bits sets an upper limit for the surface fix of any medium with its value given by

$$\phi_{S_{EH}} = c\boldsymbol{\chi}/1920 \ln 2 = 1.3769 \times 10^{69}\ \text{cycles}_l/m^2. \quad (65a)$$

E. A Brief Summary of the Lingerdynamics and the Thermodynamics Terms

As expected lingerdynamics has LT-certainty dual terms for all the known IS-uncertainty thermodynamics terms. In this section many of these terms will be discussed with the aid of three tables. First, with the aid of Table 1 the physics LT-certainty motion and IS-uncertainty retention terms, dualities and bridges will be discussed. Second, with the aid of Table 2 the statistical physics LT-certainty lingerdynamics and IS-uncertainty thermodynamics terms, dualities and bridges will be treated. Third and last, the previously derived bridges for the UNBH and ideal gas cases plus additional related concepts and extensions are summarized and further discussed with the aid of Table 3 and Appendix B.

Table 1. Selected Physics Motion/Retention Terms, Dualities and Bridges.

LT-Certainty Motion Terms	**Bridge**	**IS-Uncertainty Retention Terms**
Vacuum		Black-Hole
Motion-Time t in SI *sec*		Retention-Space ξ in SI m^3
Motion-Space $r(t)$ in SI *m*		Retention-Time $\tau(\xi)$ in SI *sec*
Space-Dislocation Δr in SI *m*		Time- Dislocation $\Delta\tau$ in SI *sec*
Life-Time Δt in SI sec		Life-Space $\Delta\xi$ in SI m^3
$\boldsymbol{c} = 2.9979 \times 10^8$ *m/sec*	$\chi = 960\pi c^2/hG$	$\chi = 6.1123 \times 10^{63}$ sec/m^2
$G = 6.67300 \times 10^{-11}$ $m^3/kg\ sec^2$	$h = 960\pi c^2/G\,\chi$	$h = 6.626068 \times 10^{-34}$ $m^2\ kg/sec$
Mass M in SI *kg*	$O = Mc^2/\chi$	Mater O in SI $kg_R = kg\ m^5/sec^3$
Mass-Energy $E=Mc^2$ in SI *J*	$\varpi = E\,\chi$	Mater-Viscidity $\varpi = O\,\chi^2$ in SI *Pa.sec*
Speed $v = \Delta r/\Delta t$ in SI *m/sec*	$\Pi = v\chi / c$	Pace $\Pi = \Delta\tau/\Delta\xi$ in SI sec/m^3
Momentum $p=Mv$ in SI *kg.m*/sec	$\upsilon = pc$	Endurance $\upsilon = O\Pi$ in SI *Joule*
Average Force $f = \Delta p/\Delta t$ in SI *N*	$\gamma = f\ v\chi = f\ c\Pi$	Average Press $\gamma = \Delta\upsilon/\Delta\xi$ in SI *Pa*
Work $W = f\Delta r$ in SI *J*	$\Psi = W\chi$	Effort $\Psi = \gamma\Delta\tau$ in SI *Pa.sec*
Wave		Weave
Wavelength λ in SI *m*		Weavelength l in SI sec
Frequency f in SI λ cycles/sec		Surface Fix ϕ_S in l cycles_l / m^2
Wave Speed $v = \lambda f$		Weave Surface Pace $\Pi_S = l\phi_S$
Spectrum (The Frequency-Wavelength Domain)		Spread (The Fix-Weavelength Domain)
Bandwidth (Spectrum of Relevant Frequencies)		Bevywidth (Spread of Relevant Fixes)

First in Table 1 selected physics LT-motion/IS-retention terms, dualities and bridges are summarized. They are:

- A *black-hole* is the retention dual of a vacuum. While a vacuum exhibits the least resistance to the motion of matter, a black-hole exhibits the least resistance to the retention of matter.
- The *retention-space* variable ξ in SI m^3 is the retention dual of the motion-time variable t in SI *sec*. While in motion problems physical variables are often investigated assuming an ideal motion environment, i.e., a vacuum, as t varies independently of them, in retention problems physical variables are often investigated assuming an ideal retention environment, i.e., a black-hole, as ξ varies independently of them.
- The *retention-time* variable $\tau(\xi)$ in SI *sec* is the retention dual of the motion-space variable $r(t)$ in SI *m*. While r is a function of the motion-time t, τ is a function of the retention-space ξ.
- The retention *time-dislocation* $\Delta\tau$ in SI *sec* is the retention dual of the motion space-dislocation Δr in SI *m.* While Δr is the distance between two points in motion-space, $\Delta\tau$ in the distance between two points in retention-time.
- The retention *life-space* $\Delta\xi$ in SI m^3 is the retention dual of the motion life-time Δt in SI *sec.* While Δt is the motion life-time used to achieve some desired space-dislocation Δr, $\Delta\xi$ is the retention life-space used to achieve some desired time-dislocation $\Delta\tau$.
- The retention *pace of dark in a black hole* χ is the retention dual of the motion *speed of light in a vacuum* $\boldsymbol{c}$. There is a bridge equation from c to χ mediated by the Plank and gravitational constants h and G, i.e. $\chi=960\pi c^2/hG$. While in a vacuum mass-less energy movers such as photons achieve the speed of light upper limit $\boldsymbol{c}$, in a black-hole mater-less *viscidity* retainers (viscidity in SI *Pa.sec* is the retention dual of energy in SI *J*) such as *portages* (the retention dual of photons) achieve the pace of dark upper limit χ.
- The retention Plank constant h in SI m^2kg/sec is the retention dual of the motion gravitational constant $\boldsymbol{G}$ in SI $m^3/kg\ sec^2$. There is a bridge equation from G to h mediated by c and χ, it is $h=960\pi c^2/G\,\chi$.

- The retention mater O in SI $kg_R=kgm^5/sec^3$ is the retention dual of the motion mass M in SI kg. There is a bridge equation from M to O mediated by $\boldsymbol{c}$ and $\boldsymbol{\chi}$, it is $O=Mc^2/\boldsymbol{\chi}$.
- The retention mater-viscidity (or O-ϖ) equation $\varpi= O\,\boldsymbol{\chi}^2$ is the retention dual of the motion mass-energy (or M-E) equation $E=Mc^2$. The retention *viscidity* ϖ in SI $Pa.sec$ is the retention dual of motion energy E in SI J. There is a bridge from E to ϖ mediated by $\boldsymbol{\chi}$, it is $\varpi=E\,\boldsymbol{\chi}$.
- The retention *pace* $\Pi=\Delta\tau/\Delta\xi$ in SI sec/m^3 is the retention dual of the motion speed $v=\Delta r/\Delta t$ in SI m/sec. While the motion speed v is the ratio of the space-dislocation achieved, Δr, per life-time used, Δt, the pace Π is the ratio of the time-dislocation achieved, $\Delta\tau$, per life-space used, $\Delta\xi$. There is a bridge equation from v to Π mediated by the pace of dark $\boldsymbol{\chi}$ and speed of light c, it is $\Pi=v\,\boldsymbol{\chi}/c$. While the maximum achievable speed is that of pure energy such as mass-less photons in a vacuum moving at the speed of light c, the maximum achievable pace is that of pure viscidity such as mater-less portages in a UNBH retaining at the pace of dark $\boldsymbol{\chi}$.
- The retention endurance expression $\upsilon=O\Pi$ is the retention dual of the motion momentum expression $p=Mv$. The retention *endurance* υ in SI J is the retention dual of motion momentum p in SI $kg.m/sec$. There is a bridge equation from p to υ mediated by c, it is $\upsilon=pc$.
- The retention average press $\gamma=\Delta\upsilon/\Delta\xi$ is the retention dual of the motion average force $f=\Delta p/\Delta t$. The retention *press* γ in SI Pa is the retention dual of motion force f in SI N. There is a bridge equation from f to γ either mediated by c and Π, it is $\gamma= f\boldsymbol{c}\Pi$, or by v and $\boldsymbol{\chi}$, it is $\gamma= fv\,\boldsymbol{\chi}$.
- The retention *effort* expression $\Psi=\gamma\Delta\tau$ is the retention dual of the motion work expression $W=f\,\Delta r$. The retention *effort* Ψ in SI $Pa.sec$ is the retention dual of motion work W in SI J. There is a bridge equation from W to Ψ mediated by $\boldsymbol{\chi}$, it is $\Psi=W\,\boldsymbol{\chi}$.
- The retention *weave* is the retention dual of the motion wave.
- The retention weavelength l in SI sec is the retention dual of the motion wavelength λ in SI m.
- The retention fix ϕ in weavelength cycles$_l$ per SI cubic meter (or cycles$_l/m^3$) is the 'volume' retention dual of the motion frequency f in SI λ cycles/sec. ϕ_S in cycles$_l/m^2$ is the 'surface' retention version of ϕ.
- The retention weave pace expression $\Pi=l\phi$ is the 'volume' retention dual of the motion wave speed expression $v=\lambda f$. $\Pi_S=l\phi_S$ in SI sec/m^2 is the 'surface' version of $\Pi=l\phi$.
- The retention *spread* is the retention dual of the motion spectrum. While the spectrum describes the motion characteristics of matter from a frequency-wavelength domain perspective, the spread describes the retention characteristics of matter from a fix-weavelength domain perspective.
- The retention *bevywidth* is the retention dual of the motion bandwidth. While bandwidth describes the spectrum of relevant frequencies-wavelengths of matter in motion, bevywidth describes the spread of relevant fixes-weavelengths of matter in retention.

Next in Table 2 selected statistical physics terms, dualities and bridges of LT-certainty lingerdynamics and IS-uncertainty thermodynamics are stated. They are:

- The dimensionless lingerdynamics-ectropy $\boldsymbol{Z}$ is the LT-certainty dual of the dimensionless thermodynamics-entropy $\boldsymbol{S}/\ln 2k=\boldsymbol{S_k}$. While $\boldsymbol{S_k}=\boldsymbol{H}$ is characterized by 'mathematical' bit units, $\boldsymbol{Z}=\boldsymbol{K}$ is characterized by 'mathematical' bor units. There is a bridge equation from $\boldsymbol{S_k}$ to $\boldsymbol{Z}$, it is $\boldsymbol{Z}=\sqrt{\boldsymbol{S_k}}$.
- The viscidity lingerature $\ddot{L}$ in SI $Pa.sec$ is the LT-certainty dual of the energy temperature $\ddot{T}=kT$ in SI J where T is the standard definition of temperature in SI K. There is a bridge equation from $\ddot{T}$ to $\ddot{L}$, it is $\ddot{L}=\ddot{T}\boldsymbol{\chi}$.
- The hover Λ in SI $Pa.sec$ is the LT-certainty dual of the heat Q in SI J. While the heat Q spontaneously transfers from a high $\ddot{T}$, i.e. $\ddot{T}_{\text{High}}$, to a low $\ddot{T}$, i.e. $\ddot{T}_{\text{Low}}$, the hover Λ spontaneously transfers from a high $\ddot{L}$, i.e. $\ddot{L}_{\text{High}}$, to a low $\ddot{L}$, i.e. $\ddot{L}_{\text{Low}}$. There is a bridge equation from Q to Λ, it is $\Lambda=Q\boldsymbol{\chi}$.
- The dimensionless hover capacity under constant duration c_τ and constant press c_γ are the LT-certainty duals of the dimensionless heat capacity under constant volume c_V and constant pressure c_P. There is a bridge equation from c_V to c_τ, it is the equality $c_\tau= c_V$. There is also a bridge equation from c_P to c_γ, it is the equality $c_\gamma= c_P$.
- The internal viscidity $\Theta=c_\tau J_p\ddot{L}$ in SI $Pa.sec$ is the LT-certainty dual of the internal energy $U=c_V J_p\ddot{T}$ in SI J of a gas. J_p is the number of particles in the gas. There is a bridge equation from U to Θ, it is $\Theta=U\boldsymbol{\chi}$.

Table 2. Selected Statistical Physics Terms, Dualities and Bridges.

IS-Uncertainty Thermodynamics (About Work and Heat Transfer)	Bridge	LT-Certainty Lingerdynamics (About Effort and Hover Transfer)
Dimensionless Thermodynamics-Entropy $\boldsymbol{S}/\ln 2k = \boldsymbol{S}_k = \boldsymbol{H}$	$\boldsymbol{Z} = \sqrt{\boldsymbol{S}_k}$	Dimensionless Lingerdynamics-Ectropy $\boldsymbol{Z}=\boldsymbol{K}$
Energy-Temperature $\dddot{T} = kT$ in SI J	$\dddot{L} = \dddot{T}\chi$	Viscidity-Lingerature $\dddot{L}$ in SI $Pa.sec$
Heat Q in SI J	$\Lambda = Q\chi$	Hover Λ in SI $Pa.sec$
Heat Capacity Constants c_V and c_P	$c_\tau = c_V$ and $c_\gamma = c_P$	Hover Capacity Constants c_τ and c_γ
Internal Energy $U = c_V J_p \dddot{T}$ in SI J	$\Theta = U\chi$	Internal Viscidity $\Theta = c_\tau J_p \dddot{L}$ in SI $Pa.sec$
Energy-Temperature as the '$\boldsymbol{S}_k=\boldsymbol{H}$ Rate' of Change of Internal Energy Over ln2 $\dddot{T} = kT = k(\partial \boldsymbol{S}/\partial U)^{-1} = (\partial U/\partial \boldsymbol{S}_k)/\ln 2 = (\partial U/\partial \boldsymbol{H})/\ln 2$	$\dddot{L} = \dddot{T}\chi$	Viscidity-Lingerature as the '$\boldsymbol{Z}^2=\boldsymbol{K}^2$ Rate' of Change of Internal Viscidity Over ln2 $\dddot{L} = (\partial\Theta/\partial \boldsymbol{Z}^2)/\ln 2 = (\partial\Theta/\partial \boldsymbol{K}^2)/\ln 2$
Pressure Times Space-Scope (or Volume) Product Energy PV in SI $J= Pa.m^3$	$\gamma\tau = PV\chi$	Press Times Time-Stay (or Duration) Product Viscidity $\gamma\tau$ in SI $Pa.sec$
The Thermodynamics Gas Law $PV = J_p \dddot{T}$ in SI J	$\gamma\tau = PV\chi$	The Lingerdynamics Gas Law $\gamma\tau = J_p \dddot{L}$ in SI $Pa.sec$
Enthalpy Heat $\overline{H} = U + PV$	$\Xi = \overline{H}\chi$	Ecthalpy Hover $\Xi=\Theta+\gamma\tau$
Helmholtz Work $\overline{A} = U - \boldsymbol{S}T = U - \ln 2\boldsymbol{H}\dddot{T}$	$\Gamma = \overline{A}\chi$	Helmholtz Effort $\Gamma = \Theta - \ln 2\boldsymbol{Z}^2\dddot{L} = \Theta - \ln 2\boldsymbol{K}^2\dddot{L}$
Gibbs Work $\overline{G} = \overline{H} - ST = \overline{H} - \ln 2\boldsymbol{H}\dddot{T}$	$Y = \overline{G}\chi$	Gibbs Effort $Y = \Xi - \ln 2\boldsymbol{Z}^2\dddot{L} = \Xi - \ln 2\boldsymbol{K}^2\dddot{L}$
The 0th Law of Thermodynamics (About Thermal Equilibrium Among Bodies) From $\dddot{T} = kT$ Definition	$\dddot{L} = \dddot{T}\chi$	The 0th Law of Lingerdynamics (About Linger Equilibrium Among Bodies) From $\dddot{L}$ Definition
The 1st Law of Thermodynamics (About Conservation of Energy) $\Delta U = \Delta Q - \Delta W$	$\Delta\Theta = \Delta U\chi$	The 1st Law of Lingerdynamics (About Conservation of Viscidity) $\Delta\Theta = \Delta\Lambda - \Delta\Psi$
The 2nd Law of Thermodynamics (About Non-Conservation of Entropy) $\delta\boldsymbol{S} = \delta Q/T = \ln 2\, k\delta\boldsymbol{H} \geq 0$	$\delta\boldsymbol{Z} = \sqrt{\delta\boldsymbol{S}/\ln 2k}$	The 2nd Law of Lingerdynamics (About Non-Conservation of Ectropy) $\ln 2\delta\boldsymbol{Z}^2 = \delta\Lambda/\dddot{L} = \ln 2\delta\boldsymbol{K}^2 \geq 0$
The 3rd Law of Thermodynamics (About Impossibility of Zero Temperature) $T \neq 0$	$\dddot{L} = kT\chi$	The 3re Law of Lingerdynamics (About Impossibility of Zero Lingerature) $\dddot{L} \neq 0$
PV Diagram and its Cycles		$\gamma\tau$ Diagram and its Cycles
Spontaneous Heat Engines $\dddot{T}_{High} \Rightarrow \dddot{T}_{Low}$		Spontaneous Hover Engines $\dddot{L}_{High} \Rightarrow \dddot{L}_{Low}$
Non-Spontaneous Work Engines $\dddot{T}_{High} \Leftarrow \dddot{T}_{Low}$		Non-Spontaneous Effort Engines $\dddot{L}_{High} \Leftarrow \dddot{L}_{Low}$
Carnot Heat Engine Max. Efficiency $(\dddot{T}_{High} - \dddot{T}_{Low})/\dddot{T}_{High}$		Carnot Hover Engine Max. Efficiency $(\dddot{L}_{High} - \dddot{L}_{Low})/\dddot{L}_{High}$

- The equation $\ddot{L}=\left(\partial\Theta/\partial \boldsymbol{Z}^2\right)/\ln 2=\left(\partial\Theta/\partial \boldsymbol{K}^2\right)/\ln 2$ defining the viscidity-lingerature $\ddot{L}$ as the '$\boldsymbol{Z}^2$=$\boldsymbol{K}^2$ rate' of change of internal viscidity over ln2 is the LT-certainty dual of the equation $\ddot{T}=kT=k\left(\partial \boldsymbol{S}/\partial U\right)^{-1}$ $=\left(\partial U/\partial \boldsymbol{S}_k\right)/\ln 2=\left(\partial U/\partial \boldsymbol{H}\right)/\ln 2$ defining the energy-temperature $\ddot{T}$ as the '$\boldsymbol{S}_k$=$\boldsymbol{H}$ rate' of change of internal energy over ln2.
- The press times time-stay (or duration) product $\gamma\tau$ is the LT-certainty dual of the IS-uncertainty pressure times space-scope (or volume) product *PV*. There is a bridge equation from *PV* to $\gamma\tau$, it is $\gamma\tau=PV\boldsymbol{\chi}$.
- The lingerdynamics gas law $\gamma\tau=J_p\ddot{L}$ is the LT-certainty dual of the thermodynamics gas law $PV=J_p\ddot{T}$. There is a bridge from the thermodynamics to lingerdynamics gas law, it is $\gamma\tau=PV\boldsymbol{\chi}$.
- The *ecthalpy* hover $\Xi=\Theta+\gamma\tau$ in SI *Pa.sec* is the LT-certainty dual of the enthalpy heat $\overline{H}=U+PV$ in SI *J*. There is a bridge equation from $\overline{H}$ to Ξ, it is $\Xi=\overline{H}\boldsymbol{\chi}$.
- The Helmholtz effort $\Gamma=\Theta-\ln 2\boldsymbol{Z}^2\ddot{L}=\Theta-\ln 2\boldsymbol{K}^2\ddot{L}$ in SI *Pa.sec* is the LT-certainty dual of the Helmholtz work $\overline{A}=U-\boldsymbol{S}T=U-\ln 2\boldsymbol{H}\ddot{T}$ in SI *J*. There is a bridge equation from $\overline{A}$ to Γ, it is $\Gamma=\overline{A}\boldsymbol{\chi}$.
- The Gibbs effort $Y=\Xi-\ln 2\boldsymbol{Z}^2\ddot{L}=\Xi-\ln 2\boldsymbol{K}^2\ddot{L}$ in SI *Pa.sec* is the LT-certainty dual of the Gibbs work $\overline{G}=\overline{H}-ST=\overline{H}-\ln 2\boldsymbol{H}\ddot{T}$ in SI *J*. There is a bridge equation from $\overline{G}$ to *Y*, it is $Y=\overline{G}\boldsymbol{\chi}$.
- The 0^{th} law of lingerdynamics (about the linger equilibrium among bodies) is the LT-certainty dual of the 0^{th} law of thermodynamics (about the thermal equilibrium among bodies). While the 0^{th} law of thermodynamics arises from the energy temperature $\ddot{T}$ definition, the 0^{th} law of lingerdynamics arises from the viscidity lingerature $\ddot{L}$ definition. There is a bridge from $\ddot{T}$ to $\ddot{L}$, it is $\ddot{L}=\ddot{T}\boldsymbol{\chi}$.
- The 1^{st} law of lingerdynamics (about the conservation of viscidity, i.e., $\Delta\Theta=\Delta\Lambda-\Delta\Psi$) is the LT-certainty dual of the 1^{st} law of thermodynamics (about the conservation of energy, i.e., $\Delta U=\Delta Q-\Delta W$). There is a bridge equation from ΔU to $\Delta\Theta$, it is $\Delta\Theta=\Delta U\boldsymbol{\chi}$.
- The 2^{nd} law of lingerdynamics (about the non-conservation of ectropy, i.e., $\ln 2\delta Z^2=\delta\Lambda/\ddot{L}=\ln 2\delta K^2\geq 0$) is the LT-certainty dual of the 2^{nd} law of thermodynamics (about the non-conservation of entropy, i.e., $\delta S=\delta Q/T=\ln 2\, k\delta H\geq 0$). There is a bridge equation from $\delta\boldsymbol{S}$ to $\delta\boldsymbol{Z}$, it is $\delta\boldsymbol{Z}=\sqrt{\delta\boldsymbol{S}/\ln 2k}$.
- The 3^{th} law of lingerdynamics (about the impossibility of zero lingerature, i.e., $\ddot{L}\neq 0$) is the LT-certainty dual of the 3^{th} law of thermodynamics (about the impossibility of zero temperature, i.e., $T\neq 0$). There is a bridge from T to $\ddot{L}$, it is $\ddot{L}=kT\boldsymbol{\chi}$.
- The $\gamma\tau$ diagram and its associated cycles is the LT-certainty dual of the *PV* diagram and its cycles. While a clockwise movement on the *PV* diagram cycle delivers work in the $\gamma\tau$ diagram delivers effort.
- The spontaneous hover engine is the LT-certainty dual of the IS-uncertainty spontaneous heat engine. While an spontaneous heat engine is linked to a spontaneous heat transfer from a higher energy-temperature $\ddot{T}_{High}$ to a lower energy-temperature $\ddot{T}_{Low}$, an spontaneous hover engine is linked to a spontaneous hover transfer from a higher viscidity-lingerature $\ddot{L}_{High}$ to a lower viscidity-lingerature $\ddot{L}_{Low}$.
- The non-spontaneous effort engine is the LT-certainty dual of the IS-uncertainty non-spontaneous work engine (i.e., cooling and heating systems). While a non-spontaneous work engine results in a non-spontaneous heat transfer from a lower energy-temperature $\ddot{T}_{Low}$ to a higher energy-temperature $\ddot{T}_{High}$, a non-spontaneous hover engine results in a non-spontaneous hover transfer from a lower viscidity-lingerature $\ddot{L}_{Low}$ to a higher viscidity-lingerature $\ddot{L}_{High}$.
- The Carnot hover engine maximum efficiency $(\ddot{L}_{High}-\ddot{L}_{Low})/\ddot{L}_{High}$ is the LT-certainty dual of the IS-uncertainty Carnot heat engine maximum efficiency $(\ddot{T}_{High}-\ddot{T}_{Low})/\ddot{T}_{High}$.

Next Table 3 summarizes the UNBH and ideal gas statistical physics bridges between their IS-entropy and LT-ectropy terms. In Appendix B each of the entries in this table are discussed.

Table 3. Summary of the UNBH and Ideal Gas Statistical Physics Bridges.

IS-Uncertainty Entropy Terms	Bridge	LT-Certainty Ectropy Terms
UNBH Entropy Bridges $\mathbf{S}_{k,EH} = \mathbf{H}_{EH} = \phi_{S_{EH}} \mathbf{N}_{EH} = \mathbf{N}_{EH}/\mathbf{N}_{Bit}$	$\mathbf{Z}_{EH} = \sqrt{\mathbf{S}_{k,EH}}$	UNBH Ectropy Bridges $\mathbf{Z}_{EH} = \mathbf{K}_{EH} = f_{EH}\ \mathbf{A}_{EH} = \mathbf{A}_{EH}/\mathbf{A}_{Bor}$
$\mathbf{N}_{EH} = 4\pi r^2 = 4\pi\left(2GM_{EH}/c^2\right)^2$	$\mathbf{A}_{EH} = \sqrt{\pi \mathbf{N}_{EH}/4c^2}$	$\mathbf{A}_{EH} = \pi r/c = 2GM_{EH}\pi/c^3$
$\mathbf{N}_{Bit} = 1/\phi_{S_{EH}} = 4\pi\Delta r^2 = 4\pi\left(2GM_{Bit}/c^2\right)^2$	$\mathbf{A}_{Bor} = \sqrt{\pi \mathbf{N}_{Bit}/4c^2}$	$\mathbf{A}_{Bor} = 1/f_{EH} = \pi\Delta r/c = 2GM_{Bit}\pi/c^3$
$\mathbf{N}_{EH}/\mathbf{N}_{Bit} = (\mathbf{M}_{EH}/\mathbf{M}_{Bit})^2 = \tau/(r\mathbf{N}_{Bit}\boldsymbol{\chi}/3) = \tau/l_{EH}$	$\mathbf{A}_{EH}/\mathbf{A}_{Bor} = \sqrt{\mathbf{N}_{EH}/\mathbf{N}_{Bit}}$	$\mathbf{A}_{EH}/\mathbf{A}_{Bor} = \mathbf{M}_{EH}/\mathbf{M}_{Bit} = \pi r/c\mathbf{A}_{Bor} = \pi r/\lambda_{EH} = \sqrt{\tau/(r\mathbf{N}_{Bit}\boldsymbol{\chi}/3)} = \sqrt{\tau/l_{EH}}$
$l_{EH} = r\mathbf{N}_{Bit}\boldsymbol{\chi}/3$	$\lambda_{EH} = \sqrt{3\pi\,\mathbf{l}_{EH}/4\boldsymbol{\chi}\,r}$	$\lambda_{EH} = \mathbf{c}\mathbf{A}_{Bor}$
$\phi_{S_{EH}} = 1/\mathbf{N}_{Bit} = c\boldsymbol{\chi}/1920\ln 2$	$f_{EH} = \sqrt{4c^2\phi_{S_{EH}}/\pi}$	$f_{EH} = 1/\mathbf{A}_{Bor} = \sqrt{c^3\boldsymbol{\chi}/480\pi\ln 2}$
$\Pi_{S_{EH}} = \phi_{S_{EH}} l_{EH} = r\boldsymbol{\chi}/3$	$v_{EH} = \sqrt{3c^2\Pi_{S_{EH}}/\chi r}$	$v_{EH} = f_{EH}\lambda_{EH} = c$
$\mathbf{M}_{Bit}/\mathbf{M}_{EH} = \sqrt{l_{EH}/\tau}$	$\lambda_{EH}/\pi\, r = \sqrt{l_{EH}/\tau}$	$\mathbf{M}_{Bit}/\mathbf{M}_{EH} = \lambda_{EH}/\pi\, r$
Single Species Ideal Gas Entropy Bridge $\mathbf{S}_{k,IG} = \mathbf{H}_{IG} = J\log_2\left(\phi_{S_{IG}}\mathbf{N}_{IG} = \mathbf{N}_{IG}/\Delta\mathbf{N}_{IG}\right)$	$\mathbf{Z}_{IG} = \sqrt{\mathbf{S}_{k,IG}}$	Single Species Ideal Gas Ectropy Bridge $\mathbf{Z}_{IG}^2 = \mathbf{K}_{IG}^2 = J\log_2\left(f_{IG}\mathbf{A}_{IG} = \mathbf{A}_{IG}/\Delta\mathbf{A}_{IG}\right)^2$
$\mathbf{N}_{IG} = 4\pi r^2 = 4\pi\left(2GM_{IG}/v_e^2\right)^2$	$\mathbf{A}_{IG} = \sqrt{\pi\mathbf{N}_{IG}/4v^2}$	$\mathbf{A}_{IG} = \pi r/v = 2GM_{IG}\pi/v_e^2 v$
$\Delta\mathbf{N}_{IG} = 1/\phi_{S_{IG}} = 4\pi\Delta r^2 = 4\pi\left(2G\Delta M_{IG}/v_e^2\right)^2$	$\Delta\mathbf{A}_{IG} = \sqrt{\pi\Delta\mathbf{N}_{IG}/4v^2}$	$\Delta\mathbf{A}_{IG} = 1/f_{IG} = \pi\Delta r/v = 2G\Delta M_{IG}\pi/v_e^2 v$
$\mathbf{N}_{IG}/\Delta\mathbf{N}_{IG} = \left(\mathbf{M}_{IG}/\Delta\mathbf{M}_{IG}\right)^2 = \tau/(r\Delta\mathbf{N}_{IG}\Pi/3) = \tau/l_{IG}$	$\mathbf{A}_{IG}/\Delta\mathbf{A}_{IG} = \sqrt{\mathbf{N}_{IG}/\Delta\mathbf{N}_{IG}}$	$\mathbf{A}_{IG}/\Delta\mathbf{A}_{IG} = \pi r/v\Delta\mathbf{A}_{IG} = \pi r/\lambda_{IG} = \mathbf{M}_{IG}/\Delta\mathbf{M}_{IG} = \sqrt{\tau/(r\Delta\mathbf{N}_{IG}\Pi/3)} = \sqrt{\tau/l_{IG}}$
$l_{IG} = r\Delta\mathbf{N}_{IG}\Pi/3$	$\lambda_{IG} = \sqrt{3\pi\,\mathbf{l}_{IG}/4\Pi\, r}$	$\lambda_{IG} = \mathbf{v}\Delta\mathbf{A}_{IG}$
$\phi_{S_{IG}} = 1/\Delta\mathbf{N}_{IG} = rT^{c_V}e^{c_P}g(2\pi mkT)^{3/2}/3JT^{3/2}h^3$	$f_{IG} = \sqrt{4v^2\phi_{S_{IG}}/\pi}$	$f_{IG} = 1/\Delta\mathbf{A}_{IG} = \sqrt{4v^2 rT^{c_V}e^{c_P}g(2\pi mkT)^{5/2}/3\pi JT^{3/2}h^3}$
$\Pi_{S_{IG}} = \phi_{S_{IG}}\mathbf{l}_{IG} = r\Pi/3$	$v_{IG} = \sqrt{3v^2\Pi_{S_{IG}}/\Pi r}$	$v_{IG} = f_{IG}\lambda_{IG} = v$
$\Delta\mathbf{M}_{IG}/\mathbf{M}_{IG} = \sqrt{l_{IG}/\tau}$	$\lambda_{IG}/\pi\, r = \sqrt{l_{IG}/\tau}$	$\Delta\mathbf{M}_{IG}/\mathbf{M}_{IG} = \lambda_{IG}/\pi\, r$
Gibbs Theorem for Thermodynamics-Entropy of Ideal Gas With Q Different Types of Molecule Species $\mathbf{S}_{IG} = \sum_{i=1}^{Q}\mathbf{S}_{IG,i} = k\sum_{i=1}^{Q}J_i\ln\left(V_{IG}T^{c_V}e^{c_P}/J_iB\right)$ $\mathbf{B} = \sum_{i=1}^{Q}T^{3/2}X_i^3/g_i, \quad \mathbf{X}_i = h/\sqrt{2\pi m_i kT}$	$\mathbf{Z}_{IG} = \sqrt{\mathbf{S}_{IG}/\ln 2k}$	Gibbs Theorem for Lingerdynamics-Ectropy of Ideal Gas With Q Different Types of Molecule Species $\mathbf{Z}_{IG} = \sqrt{\sum_{i=1}^{Q}\mathbf{Z}_{IG,i}^2} = \sqrt{\sum_{i=1}^{Q}J_i\log_2\left(V_{IG}T^{c_V}e^{c_P}/J_iB\right)}$ $\mathbf{B} = \sum_{i=1}^{Q}T^{3/2}X_i^3/g_i, \quad \mathbf{X}_i = h/\sqrt{2\pi m_i kT}$
The Gibbs Theorem Entropy Bridge $\mathbf{S}_{k,IG} = \mathbf{H}_{IG} = \sum_{i=1}^{Q}\mathbf{H}_{IG,i} = \sum_{i=1}^{Q}J_i\log_2\left(\phi_{S_{IG},i}\mathbf{N}_{IG}\right)$ $= \sum_{i=1}^{Q}J_i\log_2\left(\mathbf{N}_{IG}/\Delta\mathbf{N}_{IG,i}\right)$	$\mathbf{Z}_{IG} = \sqrt{\mathbf{S}_{k,IG}}$	The Gibbs Theorem Ectropy Bridge $\mathbf{Z}_{IG} = \mathbf{K}_{IG} = \sqrt{\sum_{i=1}^{Q}\mathbf{K}_{IG,i}^2} = \sqrt{\sum_{i=1}^{Q}J_i\log_2\left(f_{IG,i}\mathbf{A}_{IG}\right)^2}$ $= \sqrt{\sum_{i=1}^{Q}J_i\log_2\left(\mathbf{A}_{IG}/\Delta\mathbf{A}_{IG,i}\right)^2}$
$\mathbf{N}_{IG} = 4\pi r^2 = 4\pi\left(2GM_{IG}/v_e^2\right)^2$	$\mathbf{A}_{IG} = \sqrt{\pi\mathbf{N}_{IG}/4v^2}$	$\mathbf{A}_{IG} = \pi r/v = 2GM_{IG}\pi/v_e^2 v$
$\Delta\mathbf{N}_{IG,i} = 1/\phi_{S_{IG},i} = 4\pi\Delta r_i^2 = 4\pi\left(2G\Delta M_{IG,i}/v_e^2\right)^2$	$\Delta\mathbf{A}_{IG,i} = \sqrt{\pi\Delta\mathbf{N}_{IG,i}/4v^2}$	$\Delta\mathbf{A}_{IG,i} = 1/f_{IG,i} = \pi\Delta r_i/v = 2G\Delta M_{IG,i}\pi/v_e^2 v$
$\mathbf{N}_{IG}/\Delta\mathbf{N}_{IG,i} = \left(\mathbf{M}_{IG}/\Delta\mathbf{M}_{IG,i}\right)^2 = \tau/(r\Delta\mathbf{N}_{IG,i}\Pi/3) = \tau/l_{IG,i}$	$\mathbf{A}_{IG}/\Delta\mathbf{A}_{IG} = \sqrt{\mathbf{N}_{IG}/\Delta\mathbf{N}_{IG}}$	$\mathbf{A}_{IG}/\Delta\mathbf{A}_{IG,i} = \mathbf{M}_{IG}/\Delta\mathbf{M}_{IG,i} = \pi r/v\Delta\mathbf{A}_{IG,i} = \pi r/\lambda_{IG,i} = \sqrt{\tau/(r\Delta\mathbf{N}_{IG,i}\Pi/3)} = \sqrt{\tau/l_{IG,i}}$
$\mathbf{l}_{IG,i} = r\Delta\mathbf{N}_{IG,i}\Pi/3$	$\boldsymbol{\lambda}_{IG,i} = \sqrt{3\pi\,\mathbf{l}_{IG,i}/4\Pi\, r}$	$\lambda_{IG,i} = \mathbf{v}\Delta\mathbf{A}_{IG,i}$
$\phi_{S_{IG},i} = 1/\Delta\mathbf{N}_{IG,i} = rT^{c_V}e^{c_P}/3J_i\sum_{i=1}^{Q}\left(T^{3/2}h^3/(2\pi m_i kT)^{3/2}g_i\right)$	$f_{IG,i} = \sqrt{4v^2\phi_{S_{IG},i}/\pi}$	$f_{IG,i}^2 = 4v^2 rT^{c_V}e^{c_P}/3\pi J_i\sum_{i=1}^{Q}\left(T^{3/2}h^3/(2\pi m_i kT)^{3/2}g_i\right)$
$\Pi_{S_{IG}} = \phi_{S_{IG},i}\mathbf{l}_{IG,i} = r\Pi/3$	$v_{IG} = \sqrt{3v^2\Pi_{S_{IG}}/\Pi r}$	$v_{IG} = f_{IG,i}\lambda_{IG,i} = v$
$\Delta\mathbf{M}_{IG,i}/\mathbf{M}_{IG} = \sqrt{l_{IG,i}/\tau}$	$\lambda_{IG,i}/\pi\, r = \sqrt{l_{IG,i}/\tau}$	$\Delta\mathbf{M}_{IG,i}/\mathbf{M}_{IG} = \lambda_{IG,i}/\pi\, r$

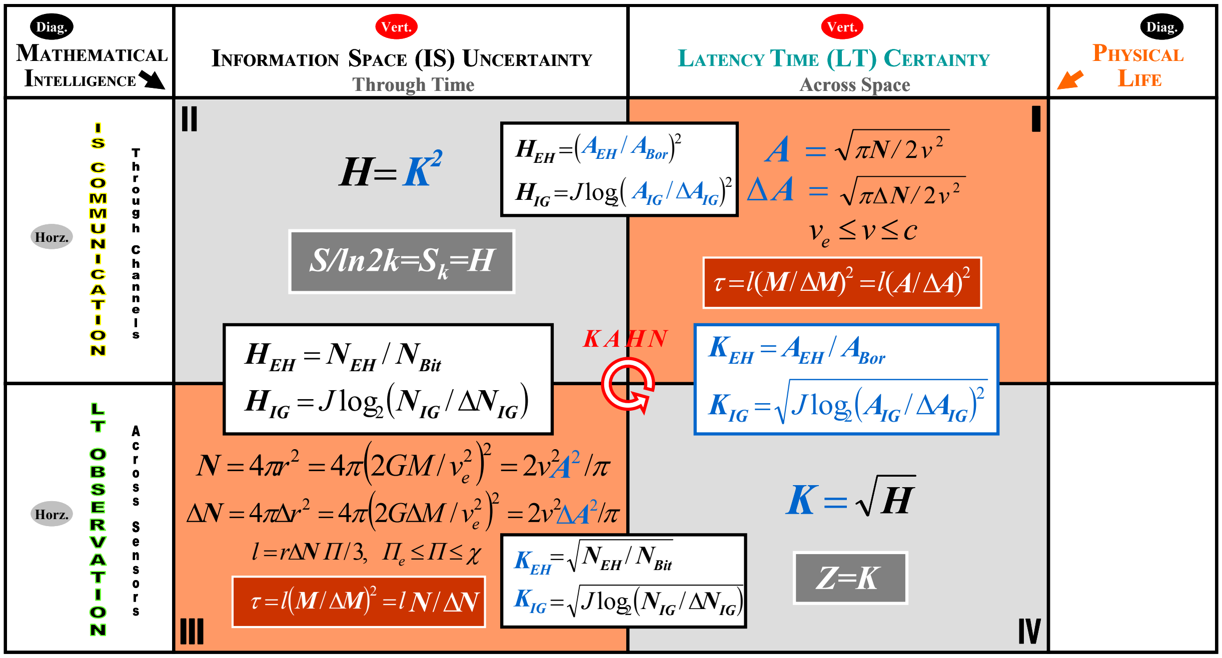

Fig. 3. The ***KAHN*** counter-clockwise statistical physics bridge sequence ***K→A→H→N*** from the LT-certainty ectropies ***K→A*** to the IS-uncertainty entropies ***H→N*** of the LIT revolution.

F. The Statistical Physics Bridges Viewed from the LIT Revolution Perspective

In Fig. 3 the principal statistical physics bridge results of the LIT revolution discussed earlier are conveniently displayed. From this figure it is first noted that the entropy/ectropy bridge relationship $A=\sqrt{\pi N/2v^2}$ of the physical-life quadrants is medium independent. This medium independence is also found to be true for the entropy/ectropy bridge relationship $K=\sqrt{H}$ of the mathematical-intelligence quadrants. On the other hand, it is also seen from Fig. 3 that when one crosses any boundary between a physical-life quadrant and a mathematical-intelligence quadrant the relationships among the entropies, ectropies or any combination of them becomes medium dependent. For instance, it is noted that while the bridge relationship between the mover-ectropy A of the physical-life quadrant I and the processor-ectropy K of the mathematical-intelligence quadrant IV is linear for a black hole since $K_{EH}=A_{EH}/A_{Bor}$, it is nonlinear for an ideal gas since $K_{IG}=\sqrt{J\log_2(A_{IG}/\Delta A_{IG})^2}$.

It is also of interest to note from Fig. 3 the medium independent quadratic relationship

$$\tau=l(M/\Delta M)^2 \qquad (66)$$

between the ratio of a retained mass M to the fractional mass ΔM that escapes it every l seconds, and the retention lifespan τ in SI seconds of M. Since (66) is medium independent it will be used in the next section to investigate the lifespan of biological systems.

4. An Illustrative Biochemistry Example

It is expected that the novel statistical physics bridge expressions summarized in Table 3 and also partially in Fig. 3 will find applications in diverse fields [11]. An interesting case that is discussed next is the study of the relationship between lifespan and daily caloric intake of biological systems. Since the human lifespan and macroscopic parameters are relatively well known, a preliminary study will be pursued for this case. For instance, the maximum human lifespan is known to be longer than 120 years where the longest unambiguously documented lifespan is that of 122 years and 164 days by Jeanne Calment of France (1875-1997). It is also well known that our cells are made mostly of water H_2O molecules with a molar mass of 18.0151 g/mol. More specifically, for a typical cell approximately 65 % of its mass is from H_2O molecules which also constitute 98.73 % of all cell molecules [12]. Furthermore, the internal temperature of our bodies is of approximately 310 K. Using H_2O molecules in our preliminary study and an assumed lifespan it will be seen that expression (66) predicts a daily caloric intake that correlates well with expectations.

The development begins by using Clausius' definition of thermodynamics-entropy [10] to model the daily digestion of food of mass ΔM with the expression

$$\Delta \boldsymbol{S}_{\rm Dig}=\Delta Q/T_{\rm Dig}=C_1C_2\Delta M/T_{\rm Dig} \text{ in SI } J/K \text{ units} \tag{67}$$

where ΔQ denotes the heat energy in SI J units of the digested food, $T_{\rm Dig}$ is the temperature of digestion in SI K units, C_1=4.2 J/cal is the amount of energy produced per calorie and C_2=5,000 $kcal/kg$ is the average amount of kilocalories per digested kilogram. The product of C_2 and ΔM, i.e. $C_2\Delta M$ thus yields the amount of kilocalories digested per day.

On the other hand, it is assumed that a matching or similar amount of mass ΔM in the form of a gas is exhaled daily by the human body. In this way the body mass M remains unaltered from day to day. Associated with this gas exhalation is the Boltzmann thermodynamic-entropy $\Delta \boldsymbol{S}_{Exh} = \boldsymbol{S}_f - \boldsymbol{S}_i$ where $\boldsymbol{S}_i$ is the thermodynamics-entropy when the day begins and $\boldsymbol{S}_f>\boldsymbol{S}_i$ is when it ends. From (49) it is noted that $\Delta \boldsymbol{S}_{Exh}$ is given by

$$\Delta \boldsymbol{S}_{Exh} = \boldsymbol{S}_f - \boldsymbol{S}_i = k(J_i+\Delta J)\ln\left(\boldsymbol{N}_{IG,f}/\Delta \boldsymbol{N}_{IG}\right) - kJ_i \ln\left(\boldsymbol{N}_{IG,i}/\Delta \boldsymbol{N}_{IG}\right)\Big|_{\boldsymbol{N}_{IG,f}=\boldsymbol{N}_{IG,i}=\boldsymbol{N}_{IG}}$$

$$= k\Delta J \ln\left(N_{IG}/\Delta N_{IG} = 3V/r\Delta N_{IG} = \tau/(r\Delta N_{IG}\Pi/3)\right) \tag{68}$$

$$N_{IG} = 4\pi r^2 = 4\pi\left(2GM/v_e^2\right)^2 \tag{69}$$

$$\Delta N_{IG} = 1/\phi_{S_{IG}} = 4\pi\Delta r^2 = 4\pi\left(2G\Delta M/v_e^2\right)^2 \tag{70}$$

$$\phi_{S_{IG}} = \sigma(em)^{5/2}\left(2\pi kT/h^2\right)^{3/2}(r/M)/3 \tag{71}$$

$$\sigma = g\, e^{c_P-5/2}T_{Exh}^{c_V-3/2} \tag{72}$$

$$m = M/J_i = 3kT/v_{rms}^2 \tag{73}$$

$$r/M = 2G/v_e^2 \tag{74}$$

$$c_V = d_f/2 \text{ and } c_P = c_V+1 \tag{75}$$

$$V=\tau/\Pi = 4\pi r^3/3 = M/1000 \tag{76}$$

where: 1) τ and Π in (68) and (76) denote lifespan in seconds and retention pace in sec/m^3, respectively; 2) the term $r\Delta N_{IG}\Pi/3$ in (68) corresponds to the time-dislocation of M or weavelength l=$r\Delta N_{IG}\Pi/3$=86,400 seconds for a single day; 3) J_i signifies the number of H_2O molecules that make up M; 4) ΔJ denotes the number of unknown particles forming the exhaled gas mass ΔM; 5) Eq. (76) assumes that the human mass density is that of liquid water, thus, for instance, if M=70 kg (154.3 lbs) then V=0.07 m^3 and r = 0.2557 m; and 6) T is the exhale temperature.

The Clausius thermodynamics-entropy (67) and the Boltzmann thermodynamics-entropy/retainer-entropy (68) expressions are next equated to yield

$$21'000{,}000\Delta M/T_{Dig} = k\Delta J \ln\left(N_{IG}/\Delta N_{IG} = \tau/86{,}400 = (M/\Delta M)^2\right). \tag{77}$$

From (77) or (66) the following relationship between the digested/exhaled (or fractional) mass ΔM and the assumed lifespan τ for a given mass M is found to be

$$\Delta M = M/\sqrt{\tau/86{,}400} = M/\sqrt{365\tau_{years}} \tag{78}$$

where $\tau_{\rm years}$ corresponds to the number of years associated with the specified τ in seconds. Notice from (78) that when the lifespan $\tau_{\rm years}$ is increased in value, the amount of daily digested/exhaled mass ΔM decreases as is expected.

The statistical physics bridge equations (68)-(78) can then be solved under different assumptions for M, τ, etc. For example, when M=70 kg, $\tau_{\rm years}$=130 (i.e. τ=4.0997 $Gsec$), T_{Dig}=T=310 K and d_f=16.1 for H_2O at 310 K, it is found that ΔM = 0.3214 kg for a *daily caloric intake of* $C_2\Delta M$=1,607 $kcal$ (other results derived from (78) with M=70 kg are $C_2\Delta M$=1,827 $kcal$ if $\tau_{\rm years}$=100, $C_2\Delta M$=2,000 $kcal$ if $\tau_{\rm years}$=83.9, etc.). The remaining operating values for $\tau_{\rm years}$=130 are: 1) σ=1.6672; 2) V=0.07 m^3; 3) r=0.2557 m; 4) N_{IG}=0.8412 m^2; 5) ΔN_{IG} =1.7311 x 10^{-5} m^2; 6) J_i=2.34 x 10^{27} H_2O molecules; 7) ΔJ=1.4643 x 10^{26} particles in ΔM with an average molar mass for ΔM of 1.3216 g/mol (e.g. this molar mass is satisfied by carbon dioxide CO_2 molecules with a total mass of 0.1736ΔM=0.0558 kg, water H_2O molecules with a total mass of 0.0714$\Delta \boldsymbol{M}$=0.0229 kg and hydrogen H atoms with a total mass of 0.755$\Delta \boldsymbol{M}$=0.2427 kg); 8) a particle escape speed of v_e=19.118 mm/sec; 9) a particle kinetic rms speed of v_{rms}=655.1496 m/sec; 10) a retention pace of Π=58.567 $Gsec/m^3$; 11) a surface fix of $\phi_{S_{IG}}$ =57.768 $kcycles_l/m^2$; and 12) a surface pace of $\Pi_{S_{IG}} = \phi_{S_{IG}}\, l$=4.9911

$Gsec/m^2$. Finally, it is noted that the previous preliminary study can be readily extended via the multi-species ideal gas model of Table 3 to more elaborate molecular models for $\boldsymbol{M}$. In Appendix C a two-species extension is given.

5. Conclusions

This paper revealed statistical physics bridges for the four quadrants of the latency information theory (LIT) revolution that included the discovery of the time dual of thermodynamics. While the physical-life quadrants I and III of the LIT revolution addressed the efficient use of the life time of physical signal movers and the life space of physical signal retainers, respectively, the mathematical-intelligence quadrants II and IV of the LIT revolution addressed the efficient use of the intelligence space of mathematical signal sources and the processing time of mathematical signal processors, respectively. Several statistical physics bridge results were derived. First, it was found that thermodynamics advanced via its thermodynamics-entropy a medium dependent bridge between the IS-uncertainty source-entropy of quadrant II and the IS-uncertainty retainer-entropy of quadrant III. Second, it was found that there is an inherent medium independent bridge between the LT-certainty mover-ectropy of quadrant I and the IS-uncertainty retainer-entropy of quadrant III. This observation then led to the discovery of the LT-certainty dual of IS-uncertainty thermodynamics that was named lingerdynamics. Lingerdynamics was then found to establish via its own lingerdyanmics-ectropy a medium dependent bridge between the LT-certainty mover-ectropy of quadrant I and the LT-certainty processor-ectropy of quadrant IV thus yielding a complete statistical physics bridge for the entropies and ectropies of the LIT revolution. For the specific cases of a UNBH medium and an ideal gas medium, complete statistical physics bridges for the LIT revolution were revealed. The paper ended with a human lifespan example that illustrated the discovery of a medium independent quadratic relationship between the lifespan of a mass in a volume and the ratio of this mass to the fractional mass that escapes it over some specified cyclic time span, e.g. the 86,400 seconds of a single day. In particular, this human example revealed for each assumed human lifespan a daily caloric intake that correlated well with expectations.

Appendix A
The Derivation of the Entropy Statistical Physics Bridge for an Ideal Gas

In this appendix the entropy statistical physics bridge expression for an ideal gas (25), i.e.,

$$\boldsymbol{S_{IG}}/\ln 2k = \boldsymbol{H_{IG}} = J\left(\ln(\boldsymbol{V_{IG}}T^{c_V}/J\mathrm{B}) + c_P\right)/\ln 2 = J\log_2\left(\boldsymbol{N_{IG}}/\Delta\boldsymbol{N_{IG}} = \tau_{IG}/l_{IG} = (\boldsymbol{M_{IG}}/\Delta\boldsymbol{M_{IG}})^2\right), \quad \text{(A.1)}$$

is derived. The derivation starts with the Boltzmann thermodynamics-entropy for an ideal gas [10]

$$\boldsymbol{S_{IG}} = kJ\left(\ln(V_{IG}T^{c_V}/JB) + c_P\right) \quad \text{(A.2)}$$

$$B = T^{3/2}\mathrm{X}^3/g \quad \text{(A.3)}$$

$$\mathrm{X} = h/\sqrt{2\pi mkT} \quad \text{(A.4)}$$

where all the variables in (A.1)-(A.4) where defined earlier in (25)-(33). Next substituting (A.4) in (A.3) one derives

$$B = h^3/g(2\pi mk)^{3/2}. \quad \text{(A.5)}$$

Next using (A.5), $V_{IG}=r\boldsymbol{N_{IG}}/3$, $m=M_{IG}/J$ and $c_P = \ln e^{c_P}$ in (A.2) it follows that

$$\boldsymbol{S_{IG}} = kJ\left(\ln(\boldsymbol{V_{IG}}T^{c_V}/JB) + c_P\right) = kJ\ln(\boldsymbol{V_{IG}}T^{c_V}e^{c_P}/JB) = kJ\ln(\phi_{S_{IG}}\boldsymbol{N_{IG}}) \quad \text{(A.6)}$$

$$\phi_{S_{IG}} = \sigma(em)^{5/2}\left(2\pi\, kT/h^2\right)^{3/2}\left(r/M_{IG}\right)/3 \quad \text{(A.7)}$$

$$\sigma = g\, e^{c_P-5/2}T^{c_V-3/2} \quad \text{(A.8)}$$

Next it is noticed that the sphere based retainer-entropy $\boldsymbol{N_{IG}}$ is given by the expression

$$\boldsymbol{N_{IG}} = 3\boldsymbol{V_{IG}}/r = 4\pi r^2 = 4\pi\left(2GM_{IG}/v_e^2\right)^2 \quad \text{(A.9)}$$

$$v_e = \sqrt{2GM_{IG}/r} \quad \text{(A.10)}$$

where v_e is the escape speed from the $\boldsymbol{N_{IG}}$ sphere that has at its center the gas mass M_{IG} that is assumed in this model to be a point-mass. Next, similarly as in (16)-(19), (24) for a UNBH the dimensionless argument $\phi_{S_{IG}}\boldsymbol{N_{IG}}$ of the natural algorithm in (A.6) is modeled as follows

$$\phi_{S_{IG}}\boldsymbol{N_{IG}} = \boldsymbol{N_{IG}}/\Delta\boldsymbol{N_{IG}} = \boldsymbol{\tau_{IG}}/\boldsymbol{l_{IG}} = (\boldsymbol{M_{IG}}/\Delta\boldsymbol{M_{IG}})^2 \quad \text{(A.11)}$$

$$\Delta \boldsymbol{N}_{IG} = 3\Delta V / \Delta r = 4\pi\Delta r^2 = 4\pi\left(2G\Delta M_{IG} / v_e^2\right)^2 \quad (A.12)$$

$$\tau_{IG} = \boldsymbol{V}_{IG}\Pi \quad (A.13)$$

$$l_{IG} = \Delta \boldsymbol{N}_{IG}\Pi\, r / 3 \quad (A.14)$$

where: 1) $\Delta\boldsymbol{N}_{IG}$ is assumed to be a small fractional retainer-entropy, i.e., $\Delta\boldsymbol{N}_{IG} << \boldsymbol{N}_{IG}$, that retains a small fractional mass ΔM_{IG}, i.e., $\Delta M_{IG} << M_{IG}$, in its sphere of radius Δr, and whose molecules can escape the gas at a speed greater than or equal to the gas escape speed v_e; 2) Π is the pace of retention in SI sec/m^3 of the ideal gas in the volume V_{IG}; 3) τ_{IG} is the lifespan of the ideal gas; and 4) l_{IG} is the cyclic time span (or weavelength) of escape from the ideal gas mass M_{IG} of the fractional mass ΔM_{IG}. Expression (A.1) then surfaces when (A.11) is substituted in (A.6) and the resulting expression for $\boldsymbol{S}_{IG}$ is substituted in $\boldsymbol{S}_{IG}/\ln 2k = \boldsymbol{H}_{IG}$.

Appendix B

The Discussion of the Statistical Physics Bridges Summarized in Table 3 for the UNBH and Ideal Gas

In this appendix the entries of Table 3 are discussed starting from the top of the table and then moving down on it:

- The UNBH ectropy bridge $\boldsymbol{Z}_{EH} = \boldsymbol{K}_{EH} = f_{EH}\boldsymbol{A}_{EH} = \boldsymbol{A}_{EH}/\boldsymbol{A}_{Bor}$ is the LT-certainty dual of the UNBH entropy bridge $\boldsymbol{S}_{k,EH} = \boldsymbol{H}_{EH} = \phi_{S_{EH}}\boldsymbol{N}_{EH} = \boldsymbol{N}_{EH}/\boldsymbol{N}_{Bit}$. The bridge from $\boldsymbol{S}_{k,EH}$ to $\boldsymbol{Z}_{EH}$ is $\boldsymbol{Z}_{EH} = \sqrt{\boldsymbol{S}_{k,EH}}$. Regarding these bridge expressions the following seven notes are made:
    1. The mover-ectropy expression $\boldsymbol{A}_{EH} = \pi r / c = 2GM_{EH}\pi / c^3$ is the LT-certainty dual of the retainer-entropy expression $\boldsymbol{N}_{EH} = 4\pi r^2 = 4\pi\left(2GM_{EH} / c^2\right)^2$. The bridge from $\boldsymbol{N}_{EH}$ to $\boldsymbol{A}_{EH}$ is $\boldsymbol{A}_{EH} = \sqrt{\pi \boldsymbol{N}_{EH} / 4c^2}$.
    2. The bor mover-ectropy expression $\boldsymbol{A}_{Bor} = 1/f_{EH} = \pi\Delta r / c = 2GM_{Bit}\pi / c^3$ is the LT-certainty dual of the bit retainer-entropy expression $\boldsymbol{N}_{Bit} = 1/\phi_{S_{EH}} = 4\pi\Delta r^2 = 4\pi\left(2GM_{Bit} / c^2\right)^2$. The bridge from $\boldsymbol{N}_{Bit}$ to $\boldsymbol{A}_{Bor}$ is $\boldsymbol{A}_{Bor} = \sqrt{\pi \boldsymbol{N}_{Bit} / 4c^2}$.
    3. The mover-ectropy ratio expression $\boldsymbol{A}_{EH}/\boldsymbol{A}_{Bor} = \boldsymbol{M}_{EH}/\boldsymbol{M}_{Bit} = \pi r/c\boldsymbol{A}_{Bor} = \pi r/\lambda_{EH} = \sqrt{\tau / (r\boldsymbol{N}_{Bit}\boldsymbol{\chi} / 3)} = \sqrt{\tau / l_{EH}}$ is the LT-certainty dual of the retainer-entropy ratio expression $\boldsymbol{N}_{EH}/\boldsymbol{N}_{Bit} = (\boldsymbol{M}_{EH}/\boldsymbol{M}_{Bit})^2 = \tau / (r\boldsymbol{N}_{Bit}\boldsymbol{\chi} / 3) = \tau / l_{EH}$. The bridge from $\boldsymbol{N}_{EH}/\boldsymbol{N}_{Bi}$ to $\boldsymbol{A}_{EH}/\boldsymbol{A}_{Bor}$ is $\boldsymbol{A}_{EH}/\boldsymbol{A}_{Bor} = \sqrt{\boldsymbol{N}_{EH}/\boldsymbol{N}_{Bit}}$.
    4. The bor wavelength expression $\lambda_{EH} = c\boldsymbol{A}_{Bor}$ is the LT-certainty dual of the bit weavelength expression $l_{EH} = r\boldsymbol{N}_{Bit}\boldsymbol{\chi}/3$. The bridge from λ_{EH} to $\boldsymbol{l}_{EH}$ is $\lambda_{EH} = \sqrt{3\pi\, \boldsymbol{l}_{EH} / 4\boldsymbol{\chi}\, r}$.
    5. The bor frequency expression $f_{EH} = 1/\boldsymbol{A}_{Bor} = \sqrt{c^3\boldsymbol{\chi} / 480\pi \ln 2}$ is the LT-certainty dual of the bit surface fix expression $\phi_{S_{EH}} = 1/\boldsymbol{N}_{Bit} = c\boldsymbol{\chi}/1920\ln 2$. The bridge from $\phi_{S_{EH}}$ to $\boldsymbol{f}_{EH}$ is $f_{EH} = \sqrt{4c^2\phi_{S_{EH}} / \pi}$.
    6. The bor velocity expression $v_{EH} = f_{EH}\lambda_{EH} = c$ is the LT-certainty dual of the bit surface pace expression $\Pi_{S_{EH}} = \phi_{S_{EH}} l_{EH} = r\boldsymbol{\chi}/3$. The bridge from $\Pi_{S_{EH}}$ to $\boldsymbol{v}_{EH}$ is $v_{EH} = \sqrt{3c^2\Pi_{S_{EH}} / \boldsymbol{\chi} r}$.
    7. The relationship $\boldsymbol{M}_{Bit} / \boldsymbol{M}_{EH} = \lambda_{EH} / \pi r$ equating the mass fraction $\boldsymbol{M}_{Bit} / \boldsymbol{M}_{EH}$ to its wavelength to space-dislocation fraction $\lambda_{EH} / \pi r$ is the LT-certainty dual of the square root relationship $\boldsymbol{M}_{BIt} / \boldsymbol{M}_{EH} = \sqrt{l_{EH} / \tau}$ relating the mass fraction $\boldsymbol{M}_{Bit} / \boldsymbol{M}_{EH}$ to its weavelength to time-dislocation squared fraction $\sqrt{l_{EH} / \tau}$. The bridge from $\boldsymbol{l}_{EH} / \tau$ to $\lambda_{EH} / \pi\, r$ is $\lambda_{EH} / \pi\, r = \sqrt{\boldsymbol{l}_{EH} / \tau}$.
- The single species ideal gas ectropy bridge $\boldsymbol{Z}_{IG}^2 = \boldsymbol{K}_{IG}^2 = J\log_2\left(f_{IG}\boldsymbol{A}_{IG} = \boldsymbol{A}_{IG}/\Delta\boldsymbol{A}_{IG}\right)^2$ is the LT-certainty dual of the single species ideal gas entropy bridge $\boldsymbol{S}_{k,IG} = \boldsymbol{H}_{IG} = J\log_2\left(\phi_{S_{IG}}\boldsymbol{N}_{IG} = \boldsymbol{N}_{IG}/\Delta\boldsymbol{N}_{IG}\right)$. The bridge from $\boldsymbol{S}_{k,IG}$ to $\boldsymbol{Z}_{IG}$ is $\boldsymbol{Z}_{IG} = \sqrt{\boldsymbol{S}_{k,IG}}$. Regarding these bridge expressions the following seven notes are made:
    1. The mover-ectropy expression $\boldsymbol{A}_{IG}^2 = (\pi r / v)^2 = \left(2GM_{IG}\pi / v_e^2 v\right)^2$ is the LT-certainty dual of the retainer-entropy expression $\boldsymbol{N}_{IG} = 4\pi r^2 = 4\pi\left(2GM_{IG} / v_e^2\right)^2$. The bridge from $\boldsymbol{N}_{IG}$ to $\boldsymbol{A}_{IG}$ is $\boldsymbol{A}_{IG} = \sqrt{\pi \boldsymbol{N}_{IG} / 4v^2}$.

2. The mover-ectropy expression $\Delta \boldsymbol{A}_{IG}^2 = 1/f_{IG}^2 = (\pi\Delta r/v)^2 = \left(2G\Delta M_{IG}\pi/v_e^2 v\right)^2$ is the LT-certainty dual of the retainer-entropy expression $\Delta \boldsymbol{N}_{IG} = 1/\phi_{S_{IG}} = 4\pi\Delta r^2 = 4\pi\left(2G\Delta M_{IG}/v_e^2\right)^2$. The bridge from $\Delta \boldsymbol{N}_{IG}$ to $\Delta \boldsymbol{A}_{IG}$ is $\Delta \boldsymbol{A}_{IG} = \sqrt{\pi\Delta \boldsymbol{N}_{IG}/4v^2}$.
3. The mover-ectropy ratio expression $\left(\boldsymbol{A}_{IG}/\Delta \boldsymbol{A}_{IG} = \pi r/v\Delta \boldsymbol{A}_{IG} = \pi r/\lambda_{IG} = \boldsymbol{M}_{IG}/\Delta \boldsymbol{M}_{IG}\right)^2 = \tau/(r\Delta \boldsymbol{N}_{IG}\Pi/3) = \tau/l_{IG}$ is the LT-certainty dual of the retainer-entropy ratio expression $\boldsymbol{N}_{IG}/\Delta \boldsymbol{N}_{IG} = \left(\boldsymbol{M}_{IG}/\Delta \boldsymbol{M}_{IG}\right)^2 = \tau/(r\Delta \boldsymbol{N}_{IG}\Pi/3) = \tau/l_{IG}$. The bridge from $\boldsymbol{N}_{IG}/\Delta \boldsymbol{N}_{IG}$ to $\boldsymbol{A}_{IG}/\Delta \boldsymbol{A}_{IG}$ is $\boldsymbol{A}_{IG}/\Delta \boldsymbol{A}_{IG} = \sqrt{\boldsymbol{N}_{IG}/\Delta \boldsymbol{N}_{IG}}$.
4. The IG wavelength expression $\lambda_{IG} = \boldsymbol{v}\Delta \boldsymbol{A}_{IG}$ is the LT-certainty dual of the IG weavelength expression $l_{IG} = r\Delta \boldsymbol{N}_{IG}\Pi/3$. The bridge from $\boldsymbol{\lambda}_{IG}$ to $\boldsymbol{l}_{IG}$ is $\lambda_{IG} = \sqrt{3\pi\, \boldsymbol{l}_{IG}/4\Pi r}$.
5. The IG frequency expression $f_{IG} = 1/\Delta \boldsymbol{A}_{IG} = \sqrt{4v^2 r T^{c_V} e^{c_P} g(2\pi mkT)^{5/2}/3\pi J T^{3/2} h^3}$ is the LT-certainty dual of the surface fix expression $\phi_{S_{IG}} = 1/\Delta \boldsymbol{N}_{IG} = r T^{c_V} e^{c_P} g(2\pi mkT)^{3/2}/3JT^{3/2}h^3$. The bridge from $\phi_{S_{IG}}$ to $\boldsymbol{f}_{IG}$ is $f_{IG} = \sqrt{4v^2\phi_{S_{IG}}/\pi}$.
6. The IG velocity expression $v_{IG} = f_{IG}\lambda_{IG} = v$ is the LT-certainty dual of the surface pace expression $\Pi_{S_{IG}} = \phi_{S_{IG}}\boldsymbol{l}_{IG} = r\Pi/3$. The bridge from $\Pi_{S_{IG}}$ to $\boldsymbol{v}_{IG}$ is $v_{IG} = \sqrt{3c^2\Pi_{S_{IG}}/\Pi r}$.
7. The relationship $\Delta \boldsymbol{M}_{IG}/\boldsymbol{M}_{IG} = \lambda_{IG}/\pi r$ equating the fractional mass fraction $\Delta \boldsymbol{M}_{IG}/\boldsymbol{M}_{IG}$ to its wavelength to space-dislocation fraction $\lambda_{IG}/\pi r$ is the LT-certainty dual of the square root relationship $\Delta \boldsymbol{M}_{IG}/\boldsymbol{M}_{IG} = \sqrt{l_{IG}/\tau}$ equating $\Delta \boldsymbol{M}_{IG}/\boldsymbol{M}_{IG}$ to its weavelength to time-dislocation fraction squared $\sqrt{l_{IG}/\tau}$. The bridge from l_{IG}/τ to $\lambda_{IG}/\pi r$ is $\lambda_{IG}/\pi r = \sqrt{l_{IG}/\tau}$.

- The third and final topic of Table 3 concerns the Gibbs theorem for the thermodynamics-entropy of an ideal gas with Q different types of species [10]. The expression for the ideal gas thermodynamics-entropy $\boldsymbol{S}_{IG}$ is given by the sum of the thermodynamics-entropy contributed by the Q species, i.e., $\boldsymbol{S}_{IG} = \sum_{i=1}^{Q}\boldsymbol{S}_{IG,i} = k\sum_{i=1}^{Q} J_i \ln\left(V_{IG}T^{c_V}e^{c_P}/J_i B\right)$ where $\boldsymbol{B} = \sum_{i=1}^{Q} T^{3/2}X_i^3/g_i$ and $\boldsymbol{X}_i = h/\sqrt{2\pi m_i kT}$. From this expression it is noted that since $\boldsymbol{S}_{IG,i} = kJ_i \ln\left(V_{IG}T^{c_V}e^{c_P}/J_i B\right)$ each species thermodynamic-entropy contribution only varies from the others if its number of molecules J_i is different. On the other hand, the ideal gas lingerdynamics-ectropy squared $\boldsymbol{Z}_{IG}^2$ is given by the sum of lingerdynamics-entropy contributed by the Q species squared, i.e., $\boldsymbol{Z}_{IG}^2 = \sum_{i=1}^{Q}\boldsymbol{Z}_{IG,i}^2 = \sum_{i=1}^{Q} J_i \log_2\left(V_{IG}T^{c_V}e^{c_P}/J_i B\right)$. The bridge from $\boldsymbol{S}_{IG}$ to $\boldsymbol{Z}_{IG}$ is given by $\boldsymbol{Z}_{IG} = \sqrt{\boldsymbol{S}_{IG}/\ln 2k}$ and from $\boldsymbol{S}_{IG,i}$ to $\boldsymbol{Z}_{G,i}$ is given by $\boldsymbol{Z}_{IG,i} = \sqrt{\boldsymbol{S}_{IG,i}/\ln 2k}$ for all i. Regarding these multi-species thermodynamic-entropy and lingerdynamics-ectropy expressions the following notes are made:
  1. The Gibbs theorem ectropy bridge $\boldsymbol{Z}_{IG}^2 = \boldsymbol{K}_{IG}^2 = \sum_{i=1}^{Q}\boldsymbol{K}_{IG,i}^2 = \sum_{i=1}^{Q} J_i \log_2\left(f_{IG,i}\boldsymbol{A}_{IG}\right)^2 = \sum_{i=1}^{Q} J_i \log_2\left(\boldsymbol{A}_{IG}/\Delta \boldsymbol{A}_{IG,i}\right)^2$ is the LT-certainty dual of the Gibbs theorem entropy bridge $\boldsymbol{S}_{k,IG} = \boldsymbol{H}_{IG} = \sum_{i=1}^{Q}\boldsymbol{H}_{IG,i} = \sum_{i=1}^{Q} J_i \log_2\left(\phi_{S_{IG},i}\boldsymbol{N}_{IG}\right) = \sum_{i=1}^{Q} J_i \log_2\left(\boldsymbol{N}_{IG,}/\Delta \boldsymbol{N}_{IG,i}\right)$. The bridge from $\boldsymbol{S}_{k,IG}$ to $\boldsymbol{Z}_{IG}$ **is** $\boldsymbol{Z}_{IG} = \sqrt{\boldsymbol{S}_{k,IG}}$.
  2. The mover-ectropy expression $\boldsymbol{A}_{IG}^2 = (\pi r/v)^2 = \left(2GM_{IG}\pi/v_e^2 v\right)^2$ is the LT-certainty dual of the retainer-entropy expression $\boldsymbol{N}_{IG} = 4\pi r^2 = 4\pi\left(2GM_{IG}/v_e^2\right)^2$. The bridge from $\boldsymbol{N}_{IG}$ to $\boldsymbol{A}_{IG}$ is $\boldsymbol{A}_{IG} = \sqrt{\pi \boldsymbol{N}_{IG}/4v^2}$.
  3. The mover-ectropy expression $\Delta \boldsymbol{A}_{IG,i}^2 = 1/f_{IG,i}^2 = (\pi\Delta r_i/v)^2 = \left(2G\Delta M_{IG,i}\pi/v_e^2 v\right)^2$ is the LT-certainty dual of the bit retainer-entropy expression $\Delta \boldsymbol{N}_{IG,i} = 1/\phi_{S_{IG},i} = 4\pi\Delta r_i^2 = 4\pi\left(2G\Delta \boldsymbol{M}_{IG,i}/v_e^2\right)^2$. The bridge from $\Delta \boldsymbol{N}_{IG,i}$ to $\Delta \boldsymbol{A}_{IG,i}$ is $\Delta \boldsymbol{A}_{IG,i} = \sqrt{\pi\Delta \boldsymbol{N}_{IG,i}/4v^2}$.

4. The mover-ectropy ratio expression $\left(A_{IG}/\Delta A_{IG,i} = \pi r/v\Delta A_{IG,i} = \pi r/\lambda_{IG,i} = M_{IG}/\Delta M_{IG,i}\right)^2 = \tau/(r\Delta N_{IG,i}\Pi/3) = \tau/l_{IG,i}$ is the LT-certainty dual of the retainer-entropy ratio expression $N_{IG}/\Delta N_{IG,i} = \left(M_{IG}/\Delta M_{IG,i}\right)^2 = \tau/(r\Delta N_{IG,i}\Pi/3) = \tau/l_{IG,i}$. The bridge from $N_{IG}/\Delta N_{IG,i}$ to $A_{IG}/\Delta A_{IG,i}$ is $A_{IG}/\Delta A_{IG,i} = \sqrt{N_{IG}/\Delta N_{IG,i}}$.
5. The IG wavelength expression $\lambda_{IG,i} = c\Delta A_{IG,i}$ is the LT-certainty dual of the IG weavelength expression $l_{IG,i} = r\Delta N_{IG,i}\Pi/3$. The bridge from $\lambda_{IG,i}$ to $l_{IG,i}$ is $\lambda_{IG,i} = \sqrt{3\pi\, l_{IG,i}/4\Pi r}$.
6. The IG frequency expression $f_{IG,i}^2 = 4v^2 rT^{c_V}e^{c_P}/3\pi J_i \sum_{i=1}^{Q}\left(T^{3/2}h^3/(2\pi m_i kT)^{3/2} g_i\right)$ is the LT-certainty dual of the surface fix expression $\phi_{S_{IG},i} = 1/\Delta N_{IG,i} = rT^{c_V}e^{c_P}/3J_i \sum_{i=1}^{Q}\left(T^{3/2}h^3/(2\pi m_i kT)^{3/2} g_i\right)$. The bridge from $\phi_{S_{IG},i}$ to $f_{IG,i}$ is $f_{IG,i} = \sqrt{4v^2\phi_{S_{IG},i}/\pi}$.
7. The IG velocity expression $v_{IG} = f_{IG,i}\lambda_{IG.i} = v$ is the LT-certainty dual of the surface pace expression $\Pi_{S_{IG}} = \phi_{S_{IG},i} l_{IG.i} = r\Pi/3$. The bridge from $\Pi_{S_{IG}}$ to v_{IG} is $v_{IG} = \sqrt{3v^2\Pi_{S_{IG}}/\Pi r}$.
8. The i^{th} molecule relationship $\Delta M_{IG,i}/M_{IG} = \lambda_{IG,i}/\pi r$ equating the fractional mass fraction $\Delta M_{IG,i}/M_{IG}$ to its wavelength to space-dislocation fraction $\lambda_{IG,i}/\pi r$ is the LT-certainty dual of the square root relationship $\Delta M_{IG,i}/M_{IG} = \sqrt{l_{IG,i}/\tau}$ equating $\Delta M_{IG,i}/M_{IG}$ to its weavelength to time-dislocation fraction squared $\sqrt{l_{IG,i}/\tau}$. The bridge from $l_{IG,i}/\tau$ to $\lambda_{IG,i}/\pi r$ is $\lambda_{IG,i}/\pi r = \sqrt{l_{IG,i}/\tau}$.

Appendix C

The Extension of the Biochemistry Example of Section 4 to the Two Species Case

In this appendix the Gibbs theorem of Table 3 for multi-species is used to approximate the Boltzman thermodynamics-entropy of a human with the following two species model

$$S = S_{Water} + S_{Other} \tag{C.1}$$

$$S_{Water} = kJ_W \ln\left(N/\Delta N_W = (M/\Delta M_W)^2 = \tau/(r\Delta N_W \Pi/3) = \tau/l_W\right) \tag{C.2}$$

$$N = 4\pi r^2 = 4\pi\left(2GM/v_e^2\right)^2 \tag{C.3}$$

$$\Delta N_W = 1/\phi_{S_W} = 4\pi\Delta r_W^2 = 4\pi\left(2G\Delta M_W/v_e^2\right)^2 = 3BJ_W/rT^{c_V}e^{c_P} \tag{C.4}$$

$$B = \left(h^3/\sqrt{2\pi k}\right)\left(1/g_W\sqrt{m_W} + 1/g_O\sqrt{m_O}\right) \tag{C.5}$$

$$S_{Other} = kJ_O \ln\left(N/\Delta N_O = (M/\Delta M_O)^2 = \tau/(r\Delta N_O \Pi/3) = \tau/l_O\right) \tag{C.6}$$

$$\Delta N_O = 1/\phi_{S_O} = 4\pi\Delta r_O^2 = 4\pi\left(2G\Delta M_O/v_e^2\right)^2 = 3BJ_O/rT^{c_V}e^{c_P} \tag{C.7}$$

In particular, the two-species model assumes that 98.73% of the molecules of the human mass M is H_2O and the remaining 1.37% are of other types (in [12] a typical 20-micron human cell is noted to contain the following percentages of different molecules: 1) 98.73% for H_2O; 2) 0.74 % for other inorganic; 3) 0.475% for lipid; 4) 0.044% for other organic; 5) 0.011% for protein; 6) $3\text{x}10^{-5}$ % for RNA; and 7) $3\text{x}10^{-11}$ % for DNA). Thus the number of water molecules is

$$J_W = 0.9873J \tag{C.8}$$

and for the other molecules is

$$J_O = 0.0137J \tag{C.9}$$

where J is the total number of molecules in M. In [12] it is also stated that 65% of a typical cell mass is H_2O with 18.0151 g/mol. Thus assuming that 65% of M is H_2O and that the J to J_W relationship of (C.8) holds it follows that

$$J = 0.65\text{x}1000 N_A M/(18.0151\text{x}0.9873) = 36.545\, N_A M \tag{C.10}$$

where N_A is Avogadro's number. Next using (C.8) and (C.9) in (C.4) over (C.7) it follows that

$$\Delta N_W/\Delta N_O = \left(\Delta M_W/\Delta M_O\right)^2 = J_W/J_O = 72.0657\,. \tag{C.11}$$

From (C.11) it then follows that

$$\Delta M_W/\Delta M_O = \sqrt{J_W/J_O} = 8.4892\,. \tag{C.12}$$

Next expressing $\Delta \boldsymbol{M_W}$ and $\Delta \boldsymbol{M_O}$ in terms of the total fractional mass $\Delta \boldsymbol{M}$ of $\boldsymbol{M}$ where

$$\Delta M = \Delta M_W + \Delta M_O \tag{C.13}$$

it follows from (C.12) and (C.13) that

$$\Delta \boldsymbol{M_W} = \Delta \boldsymbol{M} /(1+1/8.4892) = 0.8946 \Delta \boldsymbol{M} \tag{C.14}$$

$$\Delta \boldsymbol{M_O} = \Delta \boldsymbol{M} /(1+8.4892) = 0.1054 \Delta \boldsymbol{M} \tag{C.15}$$

Next using (C.14), (C.15), (C.8) and (C.9) in (C.1), (C.2) and (C.6) the following two species entropy equation results

$$\boldsymbol{S} = kJ\left(\ln(\boldsymbol{M}/\Delta \boldsymbol{M})^2 + 0.9873\ln(1/0.8946)^2 + 0.0137\ln(1/0.1054)^2\right) = kJ\left(\ln(\boldsymbol{M}/\Delta \boldsymbol{M})^2 + 0.2816\right) \tag{C.16}$$

Equation (C.16) can then be used to derive the exhale entropy expression as was done for (68) in Section 4 to yield

$$\Delta \boldsymbol{S}_{Exh} = \boldsymbol{S}_f - \boldsymbol{S}_i = k\Delta J\left(\ln\left(\boldsymbol{M}/\Delta \boldsymbol{M}\right)^2 + 0.2816\right) \tag{C.17}$$

$$(\boldsymbol{M}/\Delta \boldsymbol{M})^2 = \boldsymbol{N}/\Delta \boldsymbol{N} = \tau /(r\Delta \boldsymbol{N}\Pi/3) = \tau / l \tag{C.18}$$

Finally the Boltzmann exhale entropy (C.17) is equated to the Clausius digestion entropy (67) with the weavelength l value set to 86,400 seconds for a single day to yield the digestion/exhale entropy expression

$$21'000{,}000\Delta \boldsymbol{M}/T_{Dig} = k\Delta J\left(\ln\left(\boldsymbol{N}/\Delta \boldsymbol{N} = \tau/86{,}400 = \left(\boldsymbol{M}/\Delta \boldsymbol{M}\right)^2\right) + 0.2816\right). \tag{C.19}$$

As previously done in Section 4 for the single species case the above equations can then be solved for different cases of $\boldsymbol{M}$, τ, etc. For instance, when M=70 kg, τ_{years} =130 (i.e. τ=4.0997 *Gsec*) and T_{Dig}=T=310 *K*, it is found that ΔM=0.3214 *kg* for a daily caloric intake of $C_2\Delta M$=*1,607 kcal*, l_{IG}=86.4 *ksec*, $\phi_{S_{IG}}$ =57.768 $kcycle_l/m^2$, l_W=69.147 *ksecs*, ϕ_{S_W} =72.182 $kcycle_{lo}/m^2$, l_O=0.960 *ksec*, ϕ_{S_O} =5.2 $Mcycle_{lw}/m^2$, $\Pi_{S_{IG}} = \Pi_{S_W} = \Pi_{S_O}$ =4.9911 $Gsec/m^2$, etc.

Finally, it should be noted that the two-species methodology discussed in this appendix can be readily extended to more than two species via the Gibbs theorem of Table 3. It is hoped that a multi-species model that includes most human mass constituents will eventually lead us to exhaled gas molecule quantities and forms that correlate well with expectations. Such an outcome should in turn lead us to a better understanding of human lifespan upper bounds.

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