Linger Thermo Theory

Part I: The Dynamics Dual of the Stationary Entropy/Ectropy Based Latency Information Theory

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Abstract — A statistical-physics and information-systems based linger thermo theory is advanced that is the dynamics dual of the stationary entropy-ectropy based latency information theory. It addresses operating issues of information sources, retainers, processors and movers that are contained in a closed-system, or universe, and whose solutions are enabled by a novel unifying duality language. Linger thermo theory combines a newly enhanced thermodynamics, which addresses both information-source's order and information-retainer's retention issues, with its recently discovered time dual, called lingerdynamics that is concerned with information-processor's connection and information-mover's mobility issues. The theory is a realistic predictor of wide ranging phenomena. Among these one finds: 1) that a closed-system, or universe, continuously expands; 2) that the theoretical life expectancy of an adult living system can be mass independent, an unexpected and surprising 2010 linger thermo prediction strongly supported by a lifespan study started in the 1980s of rhesus monkeys by the United States National Institute of Aging (NIA) whose results were published in a 2012 Nature article: the NIA investigators were shocked with these results since they actually aimed to show that the life expectancy of higher mass (obese) rhesus monkeys was significantly less than that of lower mass (non-obese) ones; and 3) an equation that predicts our perceived faster moving of time as we age.

Keywords — thermodynamics, lingerdynamics, entropy, ectropy, information, latency, statistical physics, lifespan, cosmology, time compression, biology, biochemistry

I. Introduction

FOUR distinct classes of information systems can be identified in the analysis and design of systems [1]. Two are space-uncertainty based, thus ruled by Heisenberg's uncertainty principle for the configuration of space [2], while the other two are time-certainty based, and thus ruled by the uniform passing of time. The two space-uncertainty based systems are: an *information-source* where *mathematical* binary digits (bits) are space-units used to describe the expected *quantity of source-information space* in each sourced outcome realization, and an *information-retainer* where *physical* SI squared meters (m^2) are space-units used to describe the expected *quantity of retention-information space* in each retained microstate [2] realization (in the form of its volume's surface area). On the other hand, the two time-certainty based systems are: an *information-processor* where *mathematical* binary operators (bors) (or gate levels) are time-units used to describe the *quantity of processor-latency time* in each processed bit realization, and finally an *information-mover* where *physical* SI seconds (sec) are time-units used to describe the *quantity of mover-latency time* in each moved object realization.

Of these four information-system types, only an information-source has a *compression-based efficiency theory* that is anchored in thermodynamics, with its four laws that drive the Universe [3]. This efficiency theory is Shannon's information-theory [4], the catalyst and forte of the information revolution, which deals with *outcomes*, e.g., the gray levels of a monochrome image's pixels and the packets of internet's switching systems, which are specific configurations of a stochastic system that have a probability of occurrence. In contrast, in statistical thermodynamics one finds *microstates*, which are microscopic configurations of a thermodynamics system that are occupied with a certain probability in the course of thermal fluctuations. Information-theory has at its core the *source-entropy* with symbol *H* and *mathematical* bit space-units, which is defined as the *expected source-information space* of a source. This source-entropy has been used to guide the analysis and design of compression schemes of source-information space, e.g., the subband image compression methods of [5]-[6]. The success of information-theory has motivated the search for similar compression-based efficiency theories for information systems other than information-sources. In this search a duality study of joint *digital-communication* and *quantized-control* system analysis and designs of the late 1970's has been recruited. More specifically, the study started in 1978 when as part of Ph.D. dissertation investigations [7] an uncertainty-space/certainty-time duality in physics was identified in Linear Quadratic Gaussian (LQG) *continuous-control* [8]-[9]. The identification consisted of noticing that the design methodology of LQG continuous-control inherently led to the separation of the design process into separate stochastic estimator and deterministic controller design schemes that were, in turn, based in the solution of identical structure Riccati design equations. This insight was then applied to *quantized-control* to yield "Matched-Processors", the deterministic control dual of the stochastic Matched-Filters of digital-communications [10]. Matched-Processors is a practical parallel processing quantized-control scheme that does not suffer of what Bellman called, 'the curse of dimensionally' of his Dynamic Programming [11], also used in quantized-control. The employment of this novel unifying duality approach has in turn led to a nascent latency-information-theory (LIT) [1] characterized by both its efficiency and power in problem solving. For instance, in the case of radar applications [12] it has led to *knowledge-aided/unaided* adaptive radar solutions [13]-[14] subjected to taxing environmental conditions, whose signal to interference plus noise ratio (SINR) matches that of more complex *knowledge-aided* adaptive radar systems [15]. Another important finding has been the discovery [1] of a novel type of entropy characterizing information-retainers called the *retainer-entropy,* with symbol *N* and *physical* m^2 space-units, which is defined as the *expected retainer-information space* of a retainer.

More specifically, N is the expected smallest surface area amount that can enclose the information-retainer's volume, this condition occurs when the retainer's volume shape is reorganized into a sphere. N is noted to be the retainer dual of a source's H which is, in turn, the expected smallest possible number of bits that can be used to represent source-information [1], [4]. This result has given rise to the enhancement of information-theory, thus it now consists of two theories. One is 'mathematical information-theory (MIT)', where mathematical refers to the 'mathematical' bit units of source-information, thus it is the same as information-theory. The other is 'physical information-theory (PIT)', where physical refers to the 'physical' m^2 units of the retainer-information.

Another important duality result has been the emergence of the time dual for information-theory which has been called *latency-theory*. In this theory the time dual of entropy is ectropy. While the 'en' in entropy relates to looking *inside* information sources and retainers for information-space properties, the 'ec' in ectropy relates to looking *outside* information processors and movers for input-to-output latency-time properties. Using this duality approach an information-processor has been found to possess a *processor-ectropy* with symbol K and *mathematical* bor time-units. K is a minmax measure that is defined as the number of bors of latency-time, among all the processor's latency-times, that yields the maximum number of bors where each processor's latency-time denotes the smallest possible number of bors leading to a processed bit. On the other hand, an information-mover possesses a mover-ectropy with symbol A and *physical sec* time-units. A is also a minimax measure defined as the amount of seconds of latency-time, among all the mover's latency-times, that yields the maximum number of seconds where each mover's latency-time denotes the smallest possible number of seconds leading to a moved object. Similarly to our enhanced information-theory there are two special cases for latency-theory. One case is 'mathematical latency theory (MLT)', where mathematical refers to the mathematical bor time-units of the processor-latency. The other is 'physical latency theory (PLT)', where physical refers to the physical second time-units of the mover-latency.

The combination of *uncertainty-space information-theory* and *certainty-time latency-theory* resulted in LIT with applications in many areas such as in radar system analysis and design [12]-[15]. Yet the purpose of this paper is to take a judicious look at the origins of information-theory in the realms of statistical physics, i.e., statistical thermodynamics, to hopefully discover a fitting duality counterpart for LIT. This is the problem solved in this paper that yields as its solution a nascent linger thermo theory which is a realistic predictor of wide ranging phenomena such as: 1) that a closed-system, or universe, continuously expands; 2) that the lifespan of an adult individual can be mass independent as expressed by a novel quadratic equation relating it to the ratio of his mass to the mass of the daily digested food, an unexpected and surprising 2010 prediction [18] supported by rhesus monkey study results reported in a 2012 journal Nature article [19]-[20]; and 3) that we perceive time to pass at a faster rate as we age.

The rest of the paper is organized as follow. First in Section II the linger thermo theory is advanced. In Section III linger thermo relations are found for three mediums. In Section IV the spatial evolution of a closed-system, or universe, is studied. In Section V the order, retention, connection and mobility of mass-energy in a closed-system are treated. In Section VI a theoretical adult lifespan equation is revealed. In Section VII a lifespan compression equation is derived. Finally in Section VIII conclusions are drawn.

II. Linger Thermo Theory

In linger thermo theory LIT's four types of information systems are now assumed to be intertwined in a closed system, or universe as is called in thermodynamics, whose volume V contains a fixed amount of mass-energy $E=Mc^2$ where E is energy, M is mass and c is the speed of light in a vacuum. In linger thermo theory as in LIT there are entropies and ectropies that guide the study of systems. Yet the two theories are quite different since LIT's entropies and ectropies are time-invariant or stationary, while in linger thermo theory they are time-varying or dynamic. Moreover in linger thermo theory, like in LIT, there are two fundamental theories. One is the *enhanced uncertainty-space thermodynamics-theory* and the other is its recently revealed time-dual [18], i.e., the *certainty-time lingerdynamics-theory*. Furthermore, like the enhanced uncertainty-space information-theory, the enhanced uncertainty-space thermodynamics-theory also has two cases. One case is the mathematical thermodynamics-theory (MTDT) (or classical/statistical thermodynamics) characterized by the *thermo source-entropy* with symbol $\widetilde{H}$ in mathematical bit space-units evaluated according to:

$$\widetilde{H} = \log_2 \Omega = S / k \ln 2 \tag{1}$$

where Ω is the number of possible microstates that the universe can assume, S is the thermodynamics entropy in SI J/K units and k is the Boltzmann's constant also in SI J/K units. The second case is the physical thermodynamics-theory (PTDT) characterized by the *thermo retainer-entropy* with symbol $\widetilde{N}$ in physical m^2 space-units evaluated according to:

$$\widetilde{N} = 4\pi r^2 = 4\pi\left(2GM / v_e^2\right)^2 = 4\pi\left(GM / v^2\right)^2 \tag{2}$$

where $4\pi r^2$ is a spherical retainer's surface area, with r being the sphere's radius, which is the smallest possible surface area for microstate mass-energy retained in a fixed volume V, M is the information-retainer's mass that is assumed to be a point-mass residing at the sphere's center, G is the gravitational constant, v_e is the escape speed of mass-energy at the volume's edge that is also inversely related to r since $r = 2GM/v_e^2$, and v is the constant rotational speed of an object in circular motion at the volume's edge that is also inversely related to r since $r = GM/v^2$ where $v = v_e / \sqrt{2}$.

Also like the *certainty-time latency-theory* the *certainty-time lingerdynamics-theory* has two components. One is the mathematical lingerdynamics-theory (MLDT) characterized by the *linger processor-ectropy* with symbol $\widetilde{K}$ in mathematical bor time-units evaluated according to [1]:

$$\widetilde{K} = \sqrt{\widetilde{H}} \tag{3}$$

where $\widetilde{H}$ denotes the expected number of bits inputted to the information-processor and $\widetilde{K}$ denotes the number of computational delay levels (or *bors*) from input to output of the processor. For instance if $\widetilde{H}$ is 10^{24} *bits*, then the number of computational levels from input to output $\widetilde{K}$ is 10^{12} *bors*. The second and last component is the physical lingerdynamics-theory (PLDT) characterized by the *linger mover-ectropy* with symbol $\widetilde{A}$ in physical sec time-units evaluated according to [1]:

$$\widetilde{A} = \pi r / v = \sqrt{\pi \widetilde{N} / 4v^2} \tag{4}$$

where $\widetilde{A}$ is the *linger mover-ectropy* that is half the period of an object's rotational motion, driven by *M,* on the surface of a sphere of radius *r* at a constant speed of rotational motion *v*.

An investigation of natural connections between the entropies and ectropies of (1)-(4) can then be performed departing from expression (1), the *motion-time speed v, in SI s/m units,* equation:

$$v = \pi r / \widetilde{A} \tag{5}$$

and its *retention-space pace* Π, *in SI s/m³ units*, dual equation:

$$\Pi = \tau / V = 3\tau / r\widetilde{N} \tag{6}$$

where: 1) τ denotes the *lifespan of the expected number of source-information's bits* (or *life-bits* in short)—stored in a closed-system's mass-energy $E=mc^2$ of spherical volume $V = 4\pi r^3/3 = r\widetilde{N}/3$—that enable the *present* observation of some physical entity. The physical entity is either of a non-living, e.g., an image [6], or living, e.g., a human, nature. Moreover, it is assumed that life-bits can only leave the closed-system as radiation, with the caveat that if their mass-energy is later restored to the system it may no longer represent life-bits. Thus if a sufficient number of enabling life-bits are emitted without replacement from the system it can then be said that the physical entity represented by these life-bits is not longer there. An image source coding example is now used to illustrate these ideas. Consider a subbands-based minimum mean square error (MMSE) predictive-transform (PT) source coder [6] that encodes a single monochrome 512x512 pixels image, say the often encoded Lena image, in seven subbands innovation-transform-coefficient vectors $\{\delta\underline{c}'_k\text{:k=1,.,7}\}$. Our closed-system would then consist of the mass-energy representing the MMSE PT source-coder plus the mass-energy representing $\{\delta\underline{c}'_k\text{:k=1,.,7}\}$. The lifespan τ of the physical entity of interest can then be defined, for instance, as the emission time of the expected number of life-bits encoding the innovations $\{\delta\underline{c}'_k\text{:k=1,.,7}\}$ whose mass-energy may vary. Clearly similar ideas can be applied to human lifespan with life-bits radiated each day from the body cells. The loss of these biological life-bits produce the effects of aging after childhood (like lost subbands of an image [5]), say after 18 years of age when it is assumed that neither *new or lost* life-bits are being *created or replaced*, by whatever means, at a satisfactory rate; and 2) Π denotes the ratio of the lifespan τ of life-bits over the volume *V* of a closed-system, and is the uncertainty-space retention dual of the certainty-time motion speed *v*. For example, an adult with V=0.07 m^3 and τ_{Max} of 102 years—with a total lifespan of 120 years, inclusive of his first 18 yrs—will age at a pace of Π=102/0.07=1,457 adult life-years/m^3.

The investigation of information-systems/statistical-physics connections among (1)-(6) started with black-holes [16] and has led since then to the discovery of enabling linger-thermo relations. The first relation of interest is '*the universal linger-thermo equation'* that advances the following natural bridge between entropies, ectropies and physical variables:

$$\widetilde{\boldsymbol{H}} = g_{Med}\left(\frac{\widetilde{\boldsymbol{N}}}{\Delta\widetilde{\boldsymbol{N}}} = \frac{\boldsymbol{V}}{\Delta\boldsymbol{V}} = \frac{\tau}{\Delta\tau} = \left(\frac{\boldsymbol{r}}{\Delta\boldsymbol{r}}\right)^2 = \left(\frac{\boldsymbol{M}}{\Delta\boldsymbol{M}}\right)^2 = \left(\frac{\widetilde{\boldsymbol{A}}}{\Delta\widetilde{\boldsymbol{A}}}\right)^2\right) = \widetilde{\boldsymbol{K}}^2 \tag{7}$$

where g_{Med} is a medium dependent function that relates the mathematical-units entropy-ectropy pair $(\widetilde{\boldsymbol{H}},\widetilde{\boldsymbol{K}})$ to: 1) the physical-units entropy-ectropy pair $(\widetilde{\boldsymbol{N}},\widetilde{\boldsymbol{A}})$ and its so-called *quantum of operation* (QOO) *version* $(\Delta\widetilde{\boldsymbol{N}} = 4\pi\Delta\boldsymbol{r}^2, \Delta\widetilde{\boldsymbol{A}} = \pi\Delta\boldsymbol{r}/\boldsymbol{v})$; and 2) the spherical-volume $V=\tau/\Pi=r\widetilde{N}/3$, lifespan τ, radius r and mass $M=rv^2/G=v^3\widetilde{A}/\pi G$ and their QOO-volume $\Delta V=\Delta\tau/\Pi=r\Delta\widetilde{N}/3$, QOO-lifespan $\Delta\tau$, QOO-radius Δr and QOO-mass $\Delta M=\Delta rv^2/G=v^3\Delta\widetilde{A}/\pi G$. In particular, the QOO-retainer-entropy $\Delta\widetilde{\boldsymbol{N}}$ is called the *breath of space*, while the QOO-mover-ectropy $\Delta\widetilde{\boldsymbol{A}}$ is called the *bell of time*. A second relation of interest is a set of three black-hole medium based equations plus a constant mass value, called the *quantum of radiation (QOR) life-bits (LB) linger-thermo equations* that are defined according to:

$$\Diamond\widetilde{H}^{LB}(\Delta\tau) = \varepsilon_{Med}\,\Diamond\widetilde{H}(\Delta\tau) = \Delta M/\Delta M_{BH} \tag{8}$$

$$\Diamond M^{LB}(\Delta\tau) = \Diamond\widetilde{H}^{LB}(\Delta\tau)\Diamond M_{BH}^{LB}(\Delta\tau_{BH})\Big|_{M_{BH}=M} = \varepsilon_{Med}\,\Diamond M(\Delta\tau) \tag{9}$$

$$\Diamond M_{BH}^{LB}(\Delta\tau_{BH}) = M_{BH}\left(1-\sqrt[3]{1-\Delta M_{BH}^2/M_{BH}^2}\right) \tag{10}$$

$$\Delta M_{BH} = 5.1152 \text{ x } 10^{-9}\,kg \tag{11}$$

where: a) $\Diamond\widetilde{H}^{LB}(\Delta\tau) = \Delta M/\Delta M_{BH}$ is the 'assumed' number of radiated life-bits during the QOO-lifespan $\Delta\tau$, with ΔM_{BH} being the black-hole's QOO-mass whose constant value (11) is derived later; b) $\Diamond\widetilde{H}^{LB}(\Delta\tau) = \varepsilon_{Med}\Diamond\widetilde{H}(\Delta\tau)$ is a *theoretical-bridge* between the total number of radiated QOR bits $\Diamond\widetilde{H}(\Delta\tau)$ during $\Delta\tau$ and $\Diamond\widetilde{H}^{LB}(\Delta\tau)$ with ε_{Med} being the *medium life-bit emissivity*—'assumed' one for a black-hole, i.e., ε_{BH}=1; c) $\Diamond M_{BH}^{LB}(\Delta\tau_{BH})$ is the *QOR*-mass of the single life-bit radiated from a black-hole of mass M_{BH} during its $\Delta\tau_{BH}$ as will be seen later; and d) $\Diamond M^{LB}(\Delta\tau)$ and $\Diamond M(\Delta\tau)$ are the *cumulative* QOR-mass associated with $\Diamond\widetilde{H}^{LB}(\Delta\tau)$ and $\Diamond\widetilde{H}(\Delta\tau)$, respectively. The symbol '$\Diamond$' has been reserved for use with QOR quantities. In later sections (7)-(11) will be used to make sensible predictions.

III. Linger Thermo Equations of Three Mediums

This section begins with the derivation of the pace of dark in a black-hole with symbol χ, which is the retention dual of the speed of light *c* in a vacuum. We then proceed to find (7) for black-hole, photon-gas and ideal-gas mediums.

A. Pace of Dark in a Black-Hole

A black-hole is the medium that offers the least resistant to the retention of mass-energy, while a vacuum is the one that offers the least resistant to the motion of mass-energy. It can thus be said that a black-hole and a vacuum form a retention-motion duality. Since *c* is the largest possible speed in nature, a retention-motion duality suggests that a similar maximum retention of mass-energy should exist for a black-hole. This duality result is the pace of dark in a black-hole χ first derived in [16] given by:

$$\Pi_{BH} = \frac{\tau_{BH}}{V_{BH}} = \chi = \frac{480c^2}{\hbar G} = 6.1203 \text{ x } 10^{63}\ s/m^3 \tag{12}$$

with all the variables and constants previously defined, except for the Planck constant *h* in SI *Js* units and its reduced value $\hbar = h/2\pi$, and where (12) was found under the assumption that all the radiated source-information bits are life-bits whose mass-energy is never replaced. To derive (12) one begins with:

$$S_{BH} = 2\pi^2 c^3 k r_{BH}^2 / hG \tag{13}$$

the thermodynamics entropy for a spherical in shape, uncharged, and non-rotating black-hole [2] of radius r_{BH} and where all the other quantities in (13) were earlier defined. For a black-hole [2] the following relationship relates its radius r_{BH} and mass-energy $E_{BH}=M_{BH}c^2$:

$$r_{BH} = \frac{2GM_{BH}}{c^2} = \frac{2GE_{BH}}{c^4} \qquad (14)$$

This expression can then be substituted in (13) to obtain the black-hole's thermal-energy kT_{BH} given by:

$$kT_{BH} = \left(\partial S_{BH} / k\partial E_{BH} \right)^{-1} = \frac{E_{BH}}{2S_{BH}/k} = \frac{\hbar c^5}{8\pi G E_{BH}} = \frac{\hbar c}{4\pi r_{BH}} \qquad (15)$$

where T_{BH} is the black-hole temperature in SI K units. Since the spherical in shape black-hole is assumed to be a perfect non-reflective black-body [2] its *radiation* black-body luminance [21] dE_{BH}/dt equation is given by:

$$-\frac{dE_{BH}}{dt} = \frac{\pi^3 r_{BH}^2 (kT_{BH})^4}{15\hbar^3 c^2} = \frac{\hbar c^{10}}{15360\pi G^2 E_{BH}^2} \qquad (16)$$

where all the quantities in (16) were earlier defined. Equation (16) is a first order non-linear differential equation whose solution $E_{BH}^3(t) = E_0^3 - (\hbar c^{10}/5120\pi G^2)t$ with t_0=0 & $E_{BH}(t=\tau_{BH})$=0 gives:

$$\tau_{BH} = 480 c^2 V_{BH} / \hbar G \qquad (17)$$

where $V_{BH}=4\pi(2GE_0/c^4)^3/3$ is the initial black-hole volume. Finally from (17) the desired result (12) follows. Moreover, using the thermo retainer-entropy (2) and the pace of dark expression (12) in (13) the following new version for (13) surfaces:

$$S_{BH} = k\frac{\chi c}{1920}\tilde{N}_{BH} \qquad (18)$$

where the 1920 integer number in this black-hole entropy expression can be easily remembered using an appealing memory aid. This is that the year 1920 marks the beginning of the decade in the 20th century when quantum mechanics was formulated. Moreover, if one now considers χ to be a newly found 'fourth' fundamental constant in physics, besides the c, h and G already known, (18) has only the two fundamental constants χ and c which is one less than the three of (13). Most interestingly, χ and c are also noted to convey extreme cases for the pace and speed of mass-energy under black-hole and vacuum conditions that are being kept apart by the black-hole's event horizon [2].

B. Black-Hole's Linger-Thermo Relations

Using (18) in the thermo source-entropy equation (1) yields for us the black-hole's universal linger-thermo equation:

$$\tilde{\boldsymbol{H}}_{BH} = \frac{\tilde{\boldsymbol{N}}_{BH}}{\Delta\tilde{\boldsymbol{N}}_{BH}} = \frac{\boldsymbol{V}_{BH}}{\Delta\boldsymbol{V}_{BH}} = \frac{\boldsymbol{\tau}_{BH}}{\Delta\boldsymbol{\tau}_{BH}} = \left(\frac{\boldsymbol{r}_{BH}}{\Delta\boldsymbol{r}_{BH}}\right)^2 = \left(\frac{\boldsymbol{M}_{BH}}{\Delta\boldsymbol{M}_{BH}}\right)^2 = \left(\frac{\tilde{\boldsymbol{A}}_{BH}}{\Delta\tilde{\boldsymbol{A}}_{BH}}\right)^2 = \tilde{\boldsymbol{K}}_{BH}^2 \qquad (19)$$

where the black-hole's breath of space $\Delta\tilde{\boldsymbol{N}}_{BH}$ is given by

$$\Delta\tilde{N}_{BH} = \frac{1920\ln 2}{\chi c} = 7.2534 \text{ x } 10^{-70}\ m^2 \qquad (20)$$

and its bell of time $\Delta\tilde{\boldsymbol{A}}_{BH}$ by

$$\Delta\tilde{A}_{BH} = \sqrt{\frac{960\ln 2\,\pi}{\chi c^3}} = 1.1259 \text{ x } 10^{-43}\ s \qquad (21)$$

The black-hole's QOO-volume, QOO-lifespan, QOO-radius and QOO-mass are then expressed by the following equations:

$$\Delta V_{BH} = \frac{r_{BH}\Delta\tilde{N}_{BH}}{3} = \frac{640\ln 2 r_{BH}}{\chi c} \qquad (22)$$

$$\Delta\tau_{BH} = \frac{r_{BH}\Delta\tilde{N}_{BH}\chi}{3} = \frac{640\ln 2 r_{BH}}{c} \qquad (23)$$

$$\Delta r_{BH} = \sqrt{\frac{\Delta\tilde{N}_{BH}}{4\pi}} = \sqrt{\frac{480\ln 2}{\pi\chi c}} = 7.5974 \text{ x } 10^{-36}\ m \qquad (24)$$

$$\Delta M_{BH} = \frac{\Delta r_{BH} c^2}{2G} = \sqrt{\frac{120\ln 2 c^3}{\pi\chi G^2}} = 5.1152 \text{ x } 10^{-9}\ kg \qquad (25)$$

The black-hole's mover-ectropy $\tilde{\boldsymbol{A}}_{BH}$ and $\Delta\tilde{\boldsymbol{A}}_{BH}$, both defined on its event horizon, can then be used to determine the black-hole's rotation-frequency $\hat{f}_{BH} = 1/2\tilde{A}_{BH}$ and the QOO-rotation-frequency $\Delta\hat{\boldsymbol{f}}_{BH} = 1/2\Delta\tilde{\boldsymbol{A}}_{BH}$ where the hat symbol '^' on f accentuates that this is a 'rotation' frequency rather than a 'radiation' frequency. Using (4) and (21) in (15) to express kT_{BH} in term of $\hat{f}_{BH} = c/\sqrt{8\pi r_{BH}}$ and $\Delta\hat{f}_{BH} = c/\sqrt{8\pi\Delta r_{BH}}$ the following *thermal-energy linger-thermo inequality* surfaces:

$$\begin{aligned} kT_{BH} &= \hbar\hat{f}_{BH}/\sqrt{2} = \ln 2\left(\hbar\Delta\hat{f}_{BH}\right)^2 / M_{BH}c^2 = 1.5246 \text{x } 10^{17}/M_{BH}c^2 \\ &\le kT_{BH,Max} = \frac{1.5246 \text{ x } 10^{17}}{M_{BH,Min}c^2} = \frac{1.5246 \text{ x } 10^{17}}{\Delta M_{BH}c^2} = 3.3163 \text{ x } 10^8\ J \end{aligned} \qquad (26)$$

$$M_{BH,Min} = \Delta M_{BH} \qquad (27)$$

where: 1) $kT_{BH} = \hbar\hat{\boldsymbol{f}}_{BH}/\sqrt{2}$ relates kT_{BH} to the rotational-frequency-energy $\hbar\hat{\boldsymbol{f}}_{BH}$; 2) $kT_{BH} = \ln 2\left(\hbar\Delta\hat{f}_{BH}\right)^2 / M_{BH}c^2$ relates kT_{BH} to the ratio of the QOO-energy $\hbar\Delta\hat{\boldsymbol{f}}_{BH}$ squared over $E_{BH}=M_{BH}c^2$; 3) $M_{BH,Min}$ is the minimum black-hole mass that is the same as the QOO-mass ΔM_{BH} (25); and 4) $kT_{BH,Max}$ denotes the maximum thermal-energy that is attained when $M_{BH} = M_{BH,Min}$.

The maximum thermal energy $kT_{BH,Max}$ can then be used to find the IPeak QOR frequency $f_{BH}^{IPeak}(kT_{BH})$ that generates the peak value for the *spectral radiance* [21] at the surface of the black-hole $I_{BH}^{Peak}(kT_{BH})$ in SI J/m^2 units. Planck's law for black-body radiation relates f_{BH} and $I_{BH}(kT_{BH})$ as follows:

$$I_{BH}(kT_{BH}) = 2hf_{BH}^3 / c^2\left(e^{hf_{BH}/kT_{BH}} - 1\right) \qquad (28)$$

with its peak value determined from

$$I_{BH}^{Peak}(kT_{BH}) = \frac{2h\left(f_{BH}^{IPeak}(kT_{BH})\right)^3}{c^2\left(e^{hf_{BH}^{IPeak}(kT_{BH})/kT_{BH}} - 1\right)} = \frac{1.545 \text{x} 10^{92}\, h}{c^2\left(e^{4.2589 \text{x} 10^{30}\, h} - 1\right)}(kT_{BH})^3 \qquad (29)$$

where

$$f_{BH}^{IPeak}(kT_{BH}) = 5.88 \text{ x } 10^7 T_{BH} = 4.2589 \text{ x } 10^{30}\ kT_{BH} \qquad (30)$$

Using $kT_{BH,Max} = 3.3163 \text{ x } 10^8\ J$ in (29) and (30) the peak spectral radiance $I_{BH}^{Peak}(kT_{BH})$ is found to be

$$I_{BH}^{Peak}(kT_{BH,Max}) = 1.4701 \text{ x } 10^{70}\ J/m^2 \qquad (31)$$

and the corresponding IPeak QOR frequency given by

$$f_{BH}^{IPeak}(kT_{BH,Max}) = 1.4124 \text{ x } 10^{39}\ Hz \qquad (32)$$

Substituting (32) in the QOR mass-energy expression $\Diamond E = \Diamond M c^2 = hf$ one finds the QOR-energy and QOR-mass:

$$\Diamond E_{BH}^{IPeak}(kT_{BH,Max}) = 0.9372 \text{ x } 10^6\ J \qquad (33)$$

$$\Diamond M_{BH}^{IPeak}(kT_{BH,Max}) = 1.0428 \text{ x } 10^{-11}\ kg \qquad (34)$$

Finally, from $\Diamond M = c^2 \Diamond r / 2G$ one finds the QOR radius associated with (34) to be:

$$\Diamond r_{BH}^{IPeak}(kT_{BH,Max}) = 1.5488 \text{ x } 10^{-38}\ m \qquad (35)$$

A black-hole's maximum QOR-frequency is next found to be:

$$f_{BH,Max} = 6.9281 \text{ x } 10^{41}\ Hz \qquad (36)$$

where (36) is the radiation frequency of a single life-bit whose QOR-mass is the largest possible. This frequency is also noted to be larger in value by a factor of 490.5 than the IPeak QOR frequency (32). The QOR-mass of a single life-bit $\Diamond M_{BH}^{LB}(\Delta\tau_{BH})$ is emitted during $\Delta\tau_{BH}$ and is related to the black-hole mass M_{BH} and QOO-mass ΔM_{BH} according to:

$$\Diamond M_{BH}^{LB}(\Delta\tau_{BH}) = M_{BH}\left(1 - \sqrt[3]{1 - \Delta M_{BH}^2 / M_{BH}^2}\right) \qquad (37)$$

where $M_{BH} \geq \Delta M_{BH}$ thus establishing condition (27). Expression (37) is derived as follows: First solving the non-linear differential luminance equation (16) and letting $t=\Delta\tau_{BH}$ we obtain:

$$E_{BH}^3(\Delta\tau_{BH}) = E_{BH}^3 - (\hbar c^{10}/5120\pi G^2)\Delta\tau_{BH} \qquad (38)$$

where $E_{BH}=M_{BH}c^2$ and $E_{BH}(\Delta\tau_{BH})=M_{BH}(\Delta\tau_{BH})c^2$ are the initial and final mass-energy of the black-hole after the duration of $\Delta\tau_{BH}$ secs. Next using (23), (25) and $r_{BH}=2GM_{BH}/c^2$ in (38) one finds:

$$M_{BH}(\Delta\tau_{BH}) = M_{BH}\sqrt[3]{1-\Delta M_{BH}^2/M_{BH}^2} \qquad (39)$$

The desired result (37) then follows when (39) is subtracted from M_{BH} to yield

$$\Diamond M_{BH}^{LB}(\Delta\tau_{BH}) = M_{BH} - M_{BH}(\Delta\tau_{BH}) = M_{BH}\left(1-\sqrt[3]{1-\Delta M_{BH}^2/M_{BH}^2}\right) \qquad (40)$$

Also, from (19) it is noted that $\widetilde{\boldsymbol{H}}_{\boldsymbol{BH}} = \tau_{BH}/\Delta\tau_{BH} = (M_{BH}/\Delta M_{BH})^2$ and with one less life-bit it is $\widetilde{\boldsymbol{H}}_{\boldsymbol{BH}} - 1 = \bar{\tau}_{BH}/\Delta\bar{\tau}_{BH} = (\tau_{BH}-\Delta\tau_{BH})/\Delta\tau_{BH} =$ $(\overline{M}_{BH}/\Delta\overline{M}_{BH})^2 = \left(\sqrt{1-\Delta M_{BH}^2/M_{BH}^2}M_{BH}/\Delta M_{BH}\right)^2$. Moreover from (23), (14) and (40) it is found that $\Delta\bar{\tau}_{BH} = \sqrt[3]{1-\Delta M_{BH}^2/M_{BH}^2}\,\Delta\tau_{BH}$ and $\bar{\tau}_{BH} = \sqrt[3]{1-\Delta M_{BH}^2/M_{BH}^2}\,(\tau_{BH}-\Delta\tau_{BH})$ where $\sqrt[3]{1-\Delta M_{BH}^2/M_{BH}^2}$ denotes a *time-compression* factor for both the QOO-lifespan $\Delta\tau_{BH}$ and the reduced lifespan $\tau_{BH}-\Delta\tau_{BH}$ after the emission of a single life-bit—it should be noted that the QOO-lifespan leading to (8), which yields $\Diamond\widetilde{H}^{LB}(\Delta\tau_{BH}) = \Diamond\widetilde{H}(\Delta\tau_{BH}) = \Delta M_{BH}/\Delta M_{BH} = 1$ bit for a black-hole with $\varepsilon_{BH}=1$, is not time-compressed, i.e., it is $\Delta\tau_{BH}$ rather than $\Delta\bar{\tau}_{BH}$. Also using (37) one finds $\overline{M}_{BH} = M_{BH} - \Diamond M_{BH}^{LB}(\Delta\tau_{BH}) = \sqrt[3]{1-\Delta M_{BH}^2/M_{BH}^2}\,M_{BH}$ and $\Delta\overline{M}_{BH} =$ $\left(1-\Delta M_{BH}^2/M_{BH}^2\right)^{-1/6}\Delta M_{BH}$, where $\left(1-\Delta M_{BH}^2/M_{BH}^2\right)^{-1/6}$ denotes a *mass-decompression* factor for the QOO-mass ΔM_{BH} that leads to $\Delta\overline{M}_{BH} \geq \Delta M_{BH}$. Furthermore, using (40) it follows that the maximum life-bit QOR-mass $\Diamond M_{BH,Max}^{LB}$ results when the black-hole mass achieves its minimum value of (27) thus yielding the *black-hole QOR-QOO limit-mass equation*:

$$\Diamond M_{BH,Max}^{LB} = M_{BH,Min} = \Delta M_{BH} = 5.1152\times10^{-9}\ kg\,. \qquad (41)$$

The limit-mass equation (41) is then used to find the maximum QOR frequency value (36) from $f_{BH,Max}^{LB} = \Diamond M_{BH,Max}^{LB}c^2/h$. Also using $\Diamond E = \Diamond Mc^2$ and $\Diamond M = c^2\Diamond r/2G$ the following maximum life-bit QOR-energy and QOR-radius values result:

$$\Diamond E_{BH,Max}^{LB} = 4.5972 \text{ x } 10^{8}\ J \qquad (42)$$

$$\Diamond r_{BH,Max}^{LB} = 7.597 \text{ x } 10^{-36}\ m\,. \qquad (43)$$

C. Photon-Gas's Linger-Thermo Relations

The entropy for a photon gas (PG) at temperature T_{PG} is:

$$S_{PG} = k\frac{4\pi^2(kT_{PG})^3}{45c^3\hbar^3}V_{PG} = k\frac{16\pi^3(kT_{PG})^3}{135c^3\hbar^3}r_{PG}^3 = k\frac{4\pi^2(kT_{PG})^3}{135c^3\hbar^3}r_{PG}\widetilde{N}_{PG} \qquad (44)$$

where: 1) the volume of the photon-gas has the least possible surface area, it is thus a sphere with radius r_{PG} and volume $V_{PG} = 4\pi r_{PG}^3/3$; 2) $\widetilde{N}_{PG} = 4\pi r_{PG}^2$ is the thermo retainer-entropy of the gas; and 3) the remaining quantities are appropriately defined. For our spherical photon-gas the relationship:

$$r_{PG} = GM_{PG}/v^2 = GE_{PG}/v^2c^2 \qquad (45)$$

relates its radius r_{PG} to the mass-energy $E_{PG}=M_{PG}c^2$ with v being rotational speed. Equation (45) is then used in (44) to yield the thermal-energy kT_{PG} of the ideal-gas according to:

$$kT_{PG} = \left(\frac{\partial S_{PG}}{k\partial E_{PG}}\right)^{-1} = \frac{E_{PG}}{3S_{PG}/k} = \frac{45c^9\hbar^3v^6}{16\pi^3(kT_{PG})^3G^3E_{PG}^2} \qquad (46)$$

Next using (44) in (1) the following photon-gas's universal linger-thermo equation surfaces:

$$\widetilde{H}_{PG} = \frac{\widetilde{N}_{PG}}{\Delta\widetilde{N}_{PG}} = \frac{V_{PG}}{\Delta V_{PG}} = \frac{\tau_{PG}}{\Delta\tau_{PG}} = \left(\frac{r_{PG}}{\Delta r_{PG}}\right)^2 = \left(\frac{M_{PG}}{\Delta M_{PG}}\right)^2 = \left(\frac{\widetilde{A}_{PG}}{\Delta\widetilde{A}_{PG}}\right)^2 = \widetilde{K}_{PG}^2 \qquad (47)$$

where the photon gas's breath of space $\Delta\widetilde{\boldsymbol{N}}_{\boldsymbol{PG}}$ is given by

$$\Delta\widetilde{N}_{PG} = 135\ln 2c^3\hbar^3/4\pi^2(kT_{PG})^3r_{PG} \qquad (48)$$

and its bell of time $\Delta\widetilde{A}_{PG}$ by

$$\Delta\widetilde{A}_{PG} = \sqrt{135\ln 2c^3\hbar^3/16\pi(kT_{PG})^3r_{PG}v^2} \qquad (49)$$

The photon-gas's QOO-volume, QOO-lifespan, QOO-radius and QOO-mass are then:

$$\Delta V_{PG} = \frac{r_{PG}\Delta\widetilde{N}_{PG}}{3} = \frac{45\ln 2c^3\hbar^3}{4\pi^2(kT_{PG})^3} \qquad (50)$$

$$\Delta\tau_{PG} = \frac{r_{PG}\Delta\widetilde{N}_{PG}\Pi}{3} = \frac{45\ln 2c^3\hbar^3\Pi}{4\pi^2(kT_{PG})^3} \qquad (51)$$

$$\Delta r_{PG} = \sqrt{\frac{\Delta\widetilde{N}_{PG}}{4\pi}} = \sqrt{\frac{135\ln 2c^3\hbar^3}{16\pi^3(kT_{PG})^3r_{PG}}} \qquad (52)$$

$$\Delta M_{PG} = \frac{\Delta r_{PG}v^2}{G} = \sqrt{\frac{135\ln 2c^3\hbar^3v^4}{16\pi^3G^2(kT_{PG})^3r_{PG}}} \qquad (53)$$

Next using equations (4), (49), (21) in finding the photon-gas' $\hat{f}_{PG} = 1/2\widetilde{A}_{PG}$ and $\Delta\hat{f}_{PG} = 1/2\Delta\widetilde{A}_{PG}$ and the black-hole's $\Delta\hat{f}_{BH} = 1/2\Delta\widetilde{A}_{BH}$, as well as the black-hole's kT_{BH} from (26), the photon-gas's thermal-energy of (46) can be expressed in mixed photon-gas and black-hole medium conditions as:

$$kT_{PG} = \sqrt[4]{90\ln 2}\,\hbar\sqrt{\hat{f}_{PG}\Delta\hat{f}_{BH}} = \sqrt[3]{270(\ln 2)^2}\sqrt[3]{\left(\hbar\sqrt{\Delta\hat{f}_{PG}\Delta\hat{f}_{BH}}\right)^4/M_{PG}c^2} \qquad (54)$$
$$= \sqrt[3]{270\ln 2}\sqrt[3]{kT_{BH}(\hbar\Delta\hat{f}_{PG})^2}$$

where: 1) $kT_{PG} = \sqrt[4]{90\ln 2}\,\hbar\sqrt{\hat{f}_{PG}\Delta\hat{f}_{BH}}$ relates kT_{PG} to the mixed photon-gas/black-hole medium energy $\hbar\sqrt{\hat{f}_{PG}\Delta\hat{f}_{BH}}$ with $\sqrt{\hat{f}_{PG}\Delta\hat{f}_{BH}}$ denoting a mixed rotation-frequency that is the geometric mean of the photon-gas's $\hat{f}_{PG}$ and black-hole's $\Delta\hat{f}_{BH}$; and 2) $kT_{PG} = \sqrt[3]{270(\ln 2)^2}\sqrt[3]{\left(\hbar\sqrt{\Delta\hat{f}_{PG}\Delta\hat{f}_{BH}}\right)^4/M_{PG}c^2}$ relates kT_{PG} to the ratio of the mixed photon-gas/black-hole medium energy $\hbar\sqrt{\Delta\hat{f}_{PG}\Delta\hat{f}_{BH}}$ raised to the 4/3 power over the photon-gas' mass-energy $E_{PG}=M_{PG}c^2$ raised to the 1/3 power; and 3) $kT_{PG} = \sqrt[3]{270\ln 2}\sqrt[3]{kT_{BH}(\hbar\Delta\hat{f}_{PG})^2}$ relates kT_{PG} to the photon-gas' $\hbar\Delta\hat{f}_{PG}$ raised to the 2/3 power and the thermal-energy of the black-hole kT_{BH} raised to the 1/3 power.

The photon-gas is also assumed to satisfy Planck's law for black-body radiation given by:

$$I_{PG}(kT_{PG}) = 2hf_{PG}^3/c^2\left(e^{hf_{PG}/kT_{PG}}-1\right) \qquad (55)$$

with its peak value of

$$I_{PG}^{Peak}(kT_{PG}) = \frac{2h\left(f_{PG}^{IPeak}(kT_{PG})\right)^3}{c^2\left(e^{hf_{PG}^{IPeak}(kT_{PG})/kT_{PG}}-1\right)} = \frac{154.5\text{x}10^{90}\,h}{\left(e^{4.2589\text{x}10^{30}\,h}-1\right)c^2}(kT_{PG})^3 \qquad (56)$$

which occurs when f_{PG} is

$$f_{PG}^{IPeak}(kT_{PG}) = 5.88 \text{ x } 10^7\,T_{PG} = 4.2589 \text{ x } 10^{30}\ kT_{PG} \qquad (57)$$

Finally, it is assumed that the number of QOR life-bits $\Diamond\widetilde{H}_{PG}^{LB}(\Delta\tau_{PG})$ radiated during $\Delta\tau_{PG}$ satisfies (8)-(11).

D. Ideal-Gas' Linger-Thermo Relations:

The entropy of an ideal-gas (IG) of mass M_{IG} and at temperature T_{IG} is given by:

$$S_{IG} = kJ\ln\left(\frac{gk^{c_V-3/2}e^{c_P}M_{IG}^{3/2}(kT_{IG})^{c_V}V_{IG}}{(2\pi)^{3/2}J^{5/2}\hbar^3}\right) = kJ\ln\left(\frac{gk^{c_V-3/2}e^{c_P}M_{IG}^{3/2}(kT_{IG})^{c_V}r_{IG}\widetilde{N}_{IG}}{3(2\pi)^{3/2}J^{5/2}\hbar^3}\right) \quad (58)$$

where: 1) the volume of the ideal-gas has the least possible surface area, it is thus a sphere with radius r_{IG} and volume $V_{IG} = 4\pi r_{IG}^3/3$; 2) $\widetilde{N}_{IG} = 4\pi r_{IG}^2$ is the ideal-gas's thermo retainer-entropy; 3) J is the number of gas atoms or molecules; 4) c_V and c_P are the dimensionless volume and pressure heat capacity constants, respectively; and 5) g is the microstate degeneracy in appropriate units, with g=1 for a monatomic gas.

For an ideal-gas the following relationship relates its radius r_{IG} to its mass-energy $E_{IG}=M_{IG}c^2$:

$$r_{IG} = GM_{IG}/v^2 = 2GE_{IG}/v^2c^2 \quad (59)$$

Expression (59) is then substituted in (58) to derive the thermal-energy kT_{IG} of the ideal-gas which is:

$$kT_{IG} = (\partial S_{IG}/k\partial E_{IG})^{-1} = 2E_{IG}/9J \quad (60)$$

Next using (58) in (1) the ideal-gas's universal linger-thermo equation surfaces:

$$\widetilde{H}_{IG} = J\log_2\left(\frac{\widetilde{N}_{IG}}{\Delta\widetilde{N}_{IG}} = \frac{V_{IG}}{\Delta V_{IG}} = \frac{\tau_{IG}}{\Delta\tau_{IG}} = \left(\frac{r_{IG}}{\Delta r_{IG}}\right)^2 = \left(\frac{M_{IG}}{\Delta M_{IG}}\right)^2 = \left(\frac{\widetilde{A}_{IG}}{\Delta\widetilde{A}_{IG}}\right)^2\right) = \widetilde{K}_{IG}^2 \quad (61)$$

where the photon gas's breath of space $\Delta\widetilde{N}_{IG}$ is given by

$$\Delta\widetilde{N}_{IG} = 3(2\pi)^{3/2}J^{5/2}\hbar^3/gk^{c_V-3/2}e^{c_P}M_{IG}^{3/2}(kT_{IG})^{c_V}r_{IG} \quad (62)$$

and its bell of time $\Delta\widetilde{A}_{IG}$ by

$$\Delta\widetilde{A}_{IG} = \sqrt{3\pi(2\pi)^{3/2}J^{5/2}\hbar^3/4gk^{c_V-3/2}e^{c_P}M_{IG}^{3/2}(kT_{IG})^{c_V}r_{IG}v^2} \quad (63)$$

The ideal-gas's QOO-volume, QOO-lifespan, QOO-radius and QOO-mass are then:

$$\Delta V_{IG} = r_{IG}\Delta N_{IG}/3 = (2\pi)^{3/2}J^{5/2}\hbar^3/gk^{c_V-3/2}e^{c_P}M_{IG}^{3/2}(kT_{IG})^{c_V} \quad (64)$$

$$\Delta\tau_{IG} = r_{IG}\Delta\widetilde{N}_{IG}\Pi/3 = \pi(2\pi)^{3/2}J^{5/2}\hbar^3\Pi/gk^{c_V-3/2}e^{c_P}M_{IG}^{3/2}(kT_{IG})^{c_V} \quad (65)$$

$$\Delta r_{IG} = \sqrt{\Delta\widetilde{N}_{IG}/4\pi} = \sqrt{3(2\pi)^{3/2}J^{5/2}\hbar^3/4\pi gk^{c_V-3/2}e^{c_P}M_{IG}^{3/2}(kT_{IG})^{c_V}r_{IG}} \quad (66)$$

$$\Delta M_{IG} = \Delta r_{IG}v^2/G = \sqrt{3(2\pi)^{3/2}J^{5/2}\hbar^3v^4/4\pi gk^{c_V-3/2}e^{c_P}G^2M_{IG}^{3/2}(kT_{IG})^{c_V}r_{IG}} \quad (67)$$

Next using equations (4), (63), (21) in finding the ideal-gas' $\hat{f}_{IG} = 1/2\widetilde{A}_{IG}$, the ideal-gas' $\Delta\hat{f}_{IG} = 1/2\Delta\widetilde{A}_{IG}$ and the black-hole's $\Delta\hat{f}_{BH} = 1/2\Delta\widetilde{A}_{BH}$ as well as the black-hole's thermal energy of (26), the ideal-gas's thermal-energy of (60) can be expressed in mixed ideal-gas and black-hole medium conditions as:

$$(kT_{IG})^{5/2+c_V} = \left(\frac{9\ln 2}{2}\frac{(\hbar\Delta\hat{f}_{BH})^2}{JkT_{BH}}\right)^{5/2+c_V} = \frac{\sqrt{2^9\pi^7}\ln 2}{81gk^{c_V-3/2}e^{c_P}}\left(\hbar\sqrt{\Delta\hat{f}_{IG}\Delta\hat{f}_{BH}}\right)^4 \quad (68)$$

with a monatomic gas (g=1, c_V=3/2 and c_P=5/2) yielding:

$$kT_{IG} = 9\ln 2(\hbar\Delta\hat{f}_{BH})^2/2JkT_{BH} = 0.9668\ \hbar\sqrt{\Delta\hat{f}_{IG}\Delta\hat{f}_{BH}} \quad (69)$$

where $\sqrt{\Delta\hat{f}_{IG}\Delta\hat{f}_{BH}}$ is the mixed QOO-rotation-frequency given by the geometric mean of $\Delta\hat{f}_{PG}$ and $\Delta\hat{f}_{BH}$. Moreover, $(kT_{IG})^{5/2+c_V} = \left(9\ln 2(\hbar\Delta\hat{f}_{BH})^2/2JkT_{BH}\right)^{5/2+c_V}$ is noted to relate the thermal-energy of the ideal-gas kT_{IG} to the black-hole's thermal-energy kT_{BH} and the QOO-energy $\hbar\Delta\hat{f}_{BH}$.

The ideal-gas is also assumed to satisfy Planck's law for black-body radiation which is given by:

$$I_{IG}(kT_{IG}) = 2hf_{IG}^3/c^2\left(e^{hf_{IG}/kT_{IG}}-1\right) \quad (70)$$

with its peak value of:

$$I_{IG}^{Peak}(kT_{IG}) = \frac{2h\left(f_{IG}^{IPeak}(kT_{IG})\right)^3}{c^2\left(e^{hf_{IG}^{IPeak}(kT_{IG})/kT_{IG}}-1\right)} = \frac{154.5\text{x}10^{90}h}{\left(e^{4.2589\text{x}10^{30}h}-1\right)c^2}(kT_{IG})^3 \quad (71)$$

which occurs when the QOR-frequency f_{IG} is given by:

$$f_{IG}^{IPeak}(kT_{IG}) = 5.88\text{ x }10^7\,T_{IG} = 4.2589\text{ x }10^{30}\ kT_{IG} \quad (72)$$

Finally, it is assumed that the number of QOR life-bits $\lozenge\widetilde{H}_{IG}^{LB}(\Delta\tau_{IG})$ radiated during $\Delta\tau_{IG}$ satisfies (8)-(11).

IV. The Universe Expansion

Cosmological observations strongly suggest that our universe was created in an explosion of maximally dense mass-energy more than 13.7 billion years ago [17]. Since then its volume has been continuously expanding, with an acceleration of this growth also confirmed recently. Many theoretical models have been advanced for the universe's spatial evolution, inclusive of some that are adverse to a continuous expansion. Yet, theoretical models aren't available where an ever expanding universe inherently surfaces from a 'first principles' thermodynamics perspective. The search for such models is highly desirable since thermodynamics' four laws, in both their classical and modern statistical physics formulations are believed to drive the Universe's evolution [3]. Fortunately linger thermo theory's retainer-entropy guides us to synergistically link the observed continuous expansion of the Universe to a new, more comprehensive thermodynamics. The link is based on the thermo retainer-entropy $\widetilde{\boldsymbol{N}}$ (2). While the mass-energy $E=Mc^2$ remains constant for the Universe, the magnitude of the escape speed v_e varies as the Universe's physical characteristics, or medium, changes with time. In particular, maximum-density black-hole and minimum-density photon-gas mediums are found to result in upper and lower magnitude bounds for v_e, respectively. While in the black-hole medium v_e attains the upper bound of the speed of light c, i.e., v_e=c, in the photon-gas case it continuously approaches the lower bound of zero. The lower bound for v_e is noted from the photon-gas' $v_e^2 = 8\pi\,kT_{PG}GM_{PG}/3c\hbar\sqrt[3]{20S_{PG}/k}$ expression that is derived from (44) when $2GM_{PG}/v_e^2$ replaces r_{PG}. The v_e in this expression tends to zero with the passing of time since the Universe's entropy S_{PG} continuously increases due to the 2nd law of thermodynamics while the thermal-energy kT_{PG} continuously decreases due to the 1st law of thermodynamics, which requires the conservation of energy. Using these bounds for v_e in the mass-radius equation $r = 2GM/v_e^2$, it is then found that the radius of a minimum surface area spherical Universe starts with the minimum black-hole radius $r_{Min} = 2GM_{BH}/c^2$ and ends with the maximum photon-gas radius $r_{Max} = 3c\hbar\sqrt[3]{20S_{PG}/k}/4\pi kT_{PG}$, where r_{Max} approaches infinity with the passing of time. These results confirm the existence in linger thermo theory of a nascent 'first principles' retainer-entropy enhanced thermodynamics that satisfactorily predicts the observed expansion of our Universe.

V. On the Universe's Order, Retention, Connection and Mobility

In the last section it was found that as S increases with the passing of time the thermo retainer-entropy $\widetilde{N}$ also increases. Thus it can be said that the *2nd law of source-thermodynamics* (accentuating the information-source origin of the 2nd law of thermodynamics) has a duality-physical-law that may be called the 2^{nd} *law of retainer-thermodynamics*. This new law tells us that similarly to S (or equivalently $\widetilde{H}$) the Universe's *thermo retainer-entropy* $\widetilde{N}$ increases with the passing of time. Moreover, the increase of both $\widetilde{H}$ and $\widetilde{N}$ imply from (3) and (4) as well as $v = v_e / \sqrt{2}$—relating the Universe mass-energy's escape speed v_e and rotation speed v—that both the *linger processor-ectropy* $\widetilde{K}$ and the *linger mover-ectropy* $\widetilde{A}$ also increase with the passing of time thus giving rise to two additional duality-physical-laws. One of these laws is the 2^{nd} *law of processor-lingerdynamics* telling us $\widetilde{K}$ continuously increases with the passing of time and the 2^{nd} *law of mover lingerdynamics* informing us the same for $\widetilde{A}$. Furthermore, since $\widetilde{H}$'s 2^{nd} law can be interpreted as representing a steady decrease in the Universe mass-energy's order, similar types of physical duality interpretations can be given to the 2^{nd} laws of $\widetilde{N}$, $\widetilde{K}$ and $\widetilde{A}$. Thus in the case of $\widetilde{N}$ its 2^{nd} law tells us that the Universe mass-energy's retention decreases with the passing of time since it becomes harder to keep mass-energy retained (v_e becomes smaller), while in the case of $\widetilde{K}$ its 2^{nd} law tells us that the Universe mass-energy's connection decreases with the passing of time since it becomes easier for mass-energy to lose its connections. Finally, in the case of $\widetilde{A}$ its 2^{nd} law tells us that the Universe mass-energy's mobility decreases with time since it becomes harder to keep mass-energy moving (v becomes smaller). Thus it has been found that linger thermo theory offers us four 2^{nd} laws that are linked to the Universe mass-energy's order, retention, connection and mobility.

VI. On Human Lifespan

Recently in a Nature journal article [19] it was reported that a United States National Institute of Aging (NIA) study with rhesus monkeys, started in the mid 1980's, had revealed that the life expectancy of higher mass (obese) monkeys is similar to that of lower mass ones. This result was very surprising and shocking to the researches that conducted the study since they actually aimed to prove that obese monkeys would have a much lower life expectancy [20]. Since one of the two pillars of linger thermo theory is the time dual of thermodynamics, i.e., lingerdynamics, it stands to reason to investigate if this nascent theory when combined with thermodynamics makes such predictions for general biological systems, particularly humans. Indeed, in 2010 [18], more than two years before the NIA study publication [19], theoretical support for this possibility surfaced from the following theoretical adult lifespan τ equation:

$$\tau = \Delta\tau\left(M / \Delta M\right)^2 \qquad (73)$$

that is contained within the universal linger-thermo equation (7) where: 1) M is the adult mass of an individual in SI kg units; 2) ΔM is the QOO-mass of the consumed food per day (e.g., 0.4 kg for a 2,000 kcal/day diet of a M=70 kg human); 3) $\Delta\tau$ is the QOO-lifespan, or duration of one day; and 4) τ is the theoretical adult lifespan, e.g., τ=83.9 years when M=70 kg and ΔM=0.4 kg. Expanding on this last example, when 18 years of childhood are added to the 83.9 adult years, the total life expectancy of the individual is 101.9 years, which is a reasonable estimate for a human whose maximum lifespan is around 120 years.

From the quadratic dependence of the theoretical adult lifespan τ on the mass to QOO-mass ratio $M/\Delta M$ of (73) it can now be seen how the linger thermo theory predicts the rhesus monkey results of different body masses yielding the same life expectation [19]-[20]. This would be the case since as long as an individual maintains the ratio $M/\Delta M$ constant he can assume different masses that would still give him the same theoretical adult lifespan τ. For instance, if the M=70 kg individual studied earlier—whose daily food consumption is of ΔM=0.4 kg (or 2,000 kcal/day) and thus has a theoretical adult lifespan of 83.9 years—gains fat resulting in M=100 kg but with his daily food consumption also increasing to ΔM=0.5714 kg (or 2,857 kcal/day), he would still have a τ of 83.9 years since his $M/\Delta M$ ratio has not changed. Thus it is concluded that linger thermo theory has once again made a prediction that is supported by experimental studies [19]-[20]. In the companion paper [22] a weight unbiased methodology for setting a life insurance premium is advanced which is based on the lifespan expression (73) and is the first practical application of linger thermo theory.

A final *theoretical prediction* follows from the application of the QOR life-bits linger-thermo equations (8)-(11) to human lifespan. First it is noted that the daily energy of thermal radiation by an adult individual is relatively close to the average energy of food consumed daily [21]. Thus, for instance, a 70 kg adult individual consuming 2,000 kcal per day (or ΔM =0.4 kg of food) can radiate daily $\Diamond E(\Delta\tau) = 8.368 \text{ x } 10^6 J$ of energy (4.184 J/cal was used as the conversion factor) [21]. A fraction of this QOR-energy and its QOR-mass is of life-bits (8) as follows:

$$\Diamond E^{LB}(\Delta\tau) = \varepsilon_{Adult}\ \Diamond E(\Delta\tau) = 8.368 \text{ x } 10^6 \varepsilon_{Adult} \qquad (74)$$

$$\Diamond M^{LB}(\Delta\tau) = \Diamond E^{LB}(\Delta\tau)/c^2 = 9.3108 \text{ x } 10^{-11} \varepsilon_{Adult} \qquad (75)$$

where ε_{Adult} is the *adult life-bit emissivity*. Using M=70 kg and ΔM=0.4 kg in (8)-(11) we then obtain:

$$\Diamond M_{BH}^{LB}(\Delta\tau_{BH})\Big|_{M_{BH}=M} = 1.2439 \text{ x } 10^{-19}\ kg \qquad (76)$$

$$\Diamond\widetilde{H}^{LB}(\Delta\tau) = \Diamond M^{LB}(\Delta\tau)/\Diamond M_{BH}^{LB}(\Delta\tau_{BH})\Big|_{M_{BH}=M} = 7.485 \text{ x } 10^8 \varepsilon_{Adult} \qquad (77)$$

$$\varepsilon_{AInd} = \Diamond\widetilde{H}^{LB}(\Delta\tau)/7.485x10^8 = \Delta M / 7.485x10^8 \Delta M_{BH} = 0.1045 \qquad (78)$$

In this *theoretical scenario* the adult life-bit emissivity (78) informs us that 10.45 % of the adult daily radiated energy would correspond to life-bits with an average daily emission of:

$$\Diamond\widetilde{H}^{LB}(\Delta\tau) = 7.485 \text{ x } 10^8 \times 0.1045\ bits = 0.306\ megabytes \qquad (79)$$

and *average total adult life-bit emission* of:

$$\Diamond\widetilde{H}^{LB}(\tau) = \Diamond\widetilde{H}^{LB}(\Delta\tau)\tau/\Delta\tau = 9.36\ gigabytes \qquad (80)$$

with $\tau/\Delta\tau = 83.9/(1/365) = 30{,}624$ denoting the total number of QOO-lifespans (or days or subbands) available to the adult individual.

VII. On The Sensed Passing of Time

Experimental data strongly supports [23] that an individual would sense each day of his life to be shorter than when he first became an adult, say at age eighteen. Linger thermo theory is found here to predict this result when its universal linger-thermo equation for an ideal gas (61) is used as a first order model for an adult to yield the following *QOO-lifespan compression* equation:

$$\Delta\tau_{Age} = {}_{18}CF_{Age}\Delta\tau_{18},\ \ 18 \le Age \le \Gamma \qquad (81)$$

$${}_{18}CF_{Age} = (\Pi_{Age}/\Pi_{18})(T_{18}/T_{Age})^{5/2+c_V} = (\tau_{Age}/\tau_{18})(V_{18}/V_{Age})(T_{18}/T_{Age})^{5/2+c_V} \qquad (82)$$

$$\tau_{18} = \Gamma - 18\ \ge\ \tau_{Age} = \Gamma - Age \qquad (83)$$

$$V_{18}\ \le\ V_{Age} \qquad (84)$$

$$\Pi_{18}=\tau_{18}/V_{18} \geq \Pi_{Age}=\tau_{Age}/V_{Age} \qquad (85)$$

$$T_{18} \geq T_{Age} \qquad (86)$$

where: 1) Γ is the maximum theoretical lifespan; 2) $\tau_{18}=\Gamma$ -18 and $\tau_{Age}=\Gamma$ -*Age* are the life expectancies of an individual when he was 18 and at his present age, e.g., if his current age is 48 yrs and Γ =120 yrs then τ_{18}=102 yrs and τ_{48}=72 yrs; 3) V_{18} and V_{Age} are the volumes at two different ages of an individual, e.g., V_{18}=0.07 m^3 and V_{48}=0.072 m^3 for a 70 kg individual at 18 and 48 yrs of age who has kept his mass constant; 4) (83) notes that an individual's life expectancy decreases with age; 5) (84) notes that an individual's mass density decreases with age, i.e., $M/V_{18} \geq M/V_{Age}$ with M assumed time invariant; 6) (85) notes that an individual's lifespan pace decreases with age; 7) (86) notes that an individual's temperature decreases with age, e.g., T_{18}=310 K and T_{48}=308 K; 8) $\Delta\tau_{Age}$ and $\Delta\tau_{18}$ are the QOO-lifespans of an individual at two different ages with the compression factor $_{18}CF_{Age}$ in (82) expressing a *QOO-lifespan compression factor*, e.g., in our running example when c_V=3/2 we have $_{18}CF_{48}$ =(72/102)(0.07/0.072)(310/308)^4=0.7043 and $\Delta\tau_{48}$=0.7043$\Delta\tau_{18}$, thus the sensed QOO-lifespan $\Delta\tau_{48}$ by our 48 years old adult is approximately 70 % of that when he was 18.

To derive (81)-(82) we first note from (60)-(62) that:

$$\tau_{Age} / \Delta\tau_{Age} = \widetilde{N}_{Age} / \Delta\widetilde{N}_{Age} \qquad (87)$$

$$\widetilde{N}_{Age} = 4\pi r_{Age}^2 \qquad (88)$$

$$\Delta\widetilde{N}_{Age} = \frac{3(2\pi)^{3/2} J_{Age}^{5/2}\hbar^3}{gk^{c_V-3/2}e^{c_P}M^{3/2}(kT_{Age})^{c_V} r_{Age}} = \frac{3(2\pi)^{3/2}2^{5/2}E^{5/2}\hbar^3}{9^{5/2}gk^{c_V-3/2}e^{c_P}M^{3/2}(kT_{Age})^{5/2+c_V} r_{Age}} \qquad (89)$$

Next dividing (87) by $\tau_{18} / \Delta\tau_{18} = \widetilde{N}_{18} / \Delta\widetilde{N}_{18}$ and solving for $\Delta\tau_{Age}$ one obtains (81)-(82) where $V_{Age} = 4\pi r_{Age}^3 / 3$ and $V_{18} = 4\pi r_{18}^3 / 3$. Thus once again linger thermo theory has made a reasonable prediction supported by experiments.

VIII. Conclusions

A nascent linger thermo theory was advanced as the dynamics dual of the stationary entropy-ectropy based latency information theory. It addressed operator issues of information sources, retainers, processors and movers that were embedded in a closed-system. The formulation of the theory was enabled by a carefully constructed, powerful and unifying time/space duality language that was harmonious with that of statistical-physics and information-systems. Most importantly, this duality language made possible the prediction of wide ranging phenomena that when viewed superficially appeared to be unrelated such as: 1) that a closed-system, or universe, continuously expands; 2) that the theoretical life expectancy of an adult living system can be mass independent, an unexpected and surprising 2010 theoretical prediction supported by life expectancy studies of rhesus monkeys conducted by researchers from the United States National Institute of Aging whose findings were reported in a 2012 Nature article; and 3) that as we age time is perceived to move at a faster rate. Moreover, linger thermo theory has synergistically merged thermodynamics, with its four physical laws that rule the evolution of a closed-system mass-energy's order, with its recently discovered time dual, i.e., lingerdynamics, while in the process generating novel duality physical laws that rule a closed-system mass-energy's retention, connection and mobility. Finally, in the second paper of this two paper series a theoretical adult lifespan equation of linger thermo theory will be used as the basis of a weight unbiased methodology for setting life insurance premiums.

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