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The Space Dual of Einstein's Relativity Advances the Theoretical Backing for this Year's Astronomical Observation of Dark Energy Residing in the Universe's Black Holes

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Abstract. *Propelled by the discovery in the late 1990s of the accelerated expansion of the universe, an unknown energy called dark-energy was hypothesized to be responsible. In February of this year a team of astronomers reported that this dark-energy resided within isolated supermassive black holes. In this paper the Space Dual of Einstein's Relativity (SDER) in physics, first published in a 2008 SPIE article, will be seen to theoretically support this finding. The SDER theory is anchored in the medium duality formed by a vacuum and a black hole, both extreme mediums for the motion and retention, respectively, of mass-energy. It will be seen that while motion's speeds have 'the speed of light in a vacuum' given by c=2.9979 x 10^8 m/sec as an upper bound that led to Einstein's relativity in 1905, retention's paces have 'the pace of dark in a black hole' given by χ =6.13 x 10^{63} sec/m^3 as an upper bound that led to the SDER in 2008 and its theoretical support for the finding by astronomers that dark-energy resides in isolated black holes.*

1. Introduction

Astronomical theories posit that the continuous expansion of our universe started around 13.8 billion years ago with a big bang, which gave this theory of creation the name Big Bang Theory, see Fig. 1. This continuous expansion was confirmed by making use of Doppler readings of light received from astronomical systems such as galaxies within clusters of them. In the later part of the 1990s a surprising discovery was made; it was that since around 7.5 billion years ago the expansion of the universe has been accelerating. In 2011 the astronomers that discovered the accelerated expansion were awarded Nobel prizes.

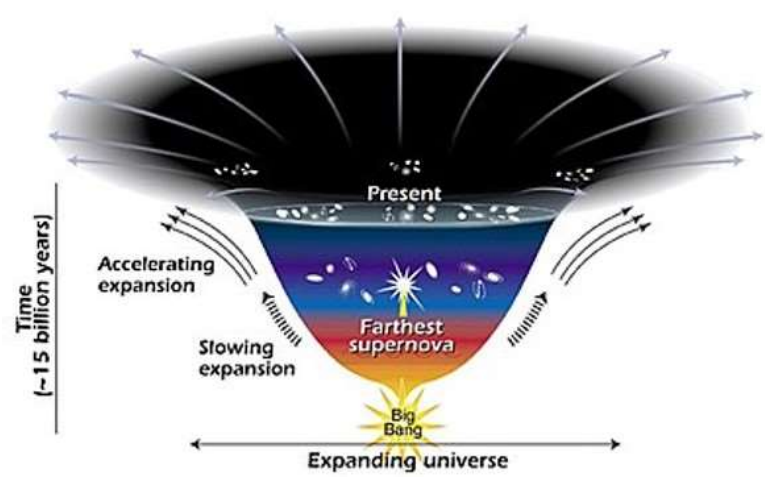

Fig. 1. The expansion of the universe

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When the acceleration of the expansion was first discovered by astronomers its source could not be identified so, they concluded that it was caused by a dark-energy that was uniformly fueled by the universe's vacuum. However, this state of affairs changed radically this year when a team of astronomers and astrophysicists found significant observational evidence that dark-energy is found inside isolated supermassive black holes as vacuum energy. Black holes continuously consume surrounding stars, gas, and other black holes which would make them larger in mass and volume, but astronomical observations have now also shown that this growth will continue even if the black holes stay in isolation, which was to astronomers an astonishing result. Because of the warping of space predicted by Einstein's general relativity, an enclosing event horizon surface area is linked to all black holes where any light crossing it can never cross it again in the opposite direction since the speed of light after crossing the event horizon is exceeded by the speed of the space warping that is simultaneously taking place. Because of this event horizon property, black holes cannot be observed directly by outside observers, thus at best only inferential artificial intelligence illustrations of black holes can be derived such as the one depicted in Fig. 2 that appeared in the American Astronomical Society (AAS) article [1] which shows a ring structure for the event horizon region.

The objective of this paper is to review strong duality theoretical backing advanced fifteen years ago [2] for this year's astronomical observation of dark energy residing in isolated supermassive black holes which was reported in two American Astronomical Society (AAS) articles this February [3]-[4]. This connection between dark energy and black holes has been referred to as 'cosmological coupling' and claimed to be supported in two ways. First by the observations reported in [3] of inactive red and dead elliptical galaxies that showed for their central black holes rates by a factor of 10 in their mass from seven billion years ago to today, illustrated in Fig. 3, something that the team of astronomers found to be particularly surprising and difficult to account for when using traditional methods. Secondly it is further reported in [4] that the found rate of mass increase for the black holes correlated quite well with the increase of the universe's volume. These two results were then used to justify their claim that cosmological coupling exists between dark energy and supermassive black holes, thus giving rise to observations such as [5]:

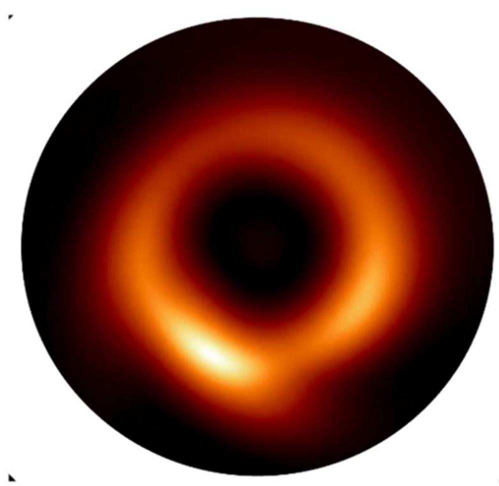

Fig. 2. An isolated supermassive black hole.

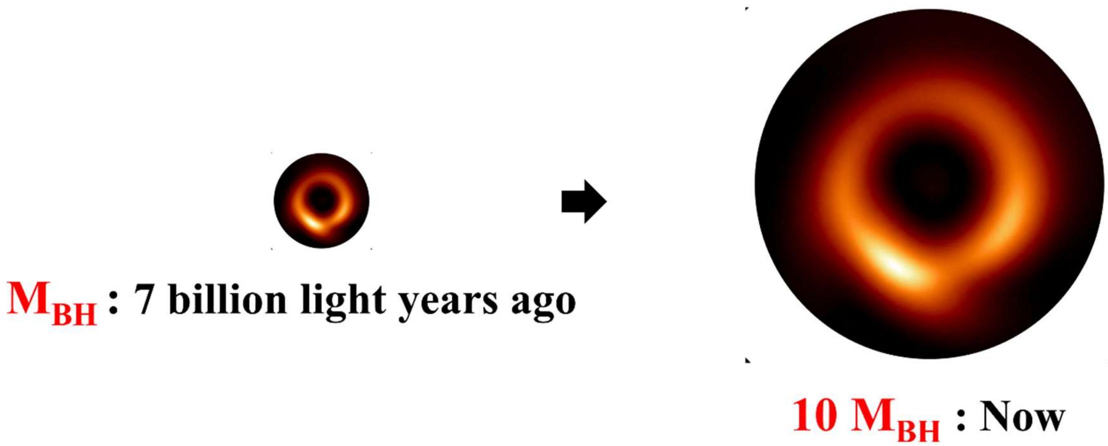

Fig. 3. An isolated supermassive black hole growth over time

- That the dark energy in the black holes is fueled by the vacuum energy found in them, thus rejecting the previous singularity hypothesis that at the center of a black hole one finds its mass in the form of an infinitely dense point mass.
- That dark energy does not need to be a property of the vacuum itself, and it does not need to be uniform. Thus, dark energy can reside within black holes, and be produced when large stars collapse in death, something that was said to have been predicted since the mid-sixties.
- That the dark energy source is provided from something that is already known to exist, namely black holes. Quite an advantage since there is no need for any new type of object or new particle.

It will be seen here that the theoretical backing for the continuous growth of isolated black holes emerged naturally in the Space Dual of Einstein's Relativity (SDER) that was first published fifteen years ago in [2]. This space dual arose naturally while guided by a fundamental past-uncertainty/future-certainty time-complementary duality principle of physics (POP) that was discovered in the late 1970s to have popped up in the 1960 Linear Quadratic Gaussian (LQG) stochastic optimum control work of Rudolph Kalman, the originator of the Kalman Filter [6]. Since its discovery POP, through its amazing, but not surprising, predictive power, has led to several innovative theories or methodologies such as Matched Processors. Matched Processors [7] provided for quantized control a simple, high performance, and affordable alternative to Dynamic Programming, that it often outperformed while also not suffering of what Bellman called 'the curse of dimensionality' for the required computations [8]. The Matched Processors method was the future-certainty dual of the past-uncertainty Matched Filters method for bit detection used in digital communications [9]. A second major theory attained through POP guidance in the mid-2000s was Latency Information Theory with important applications in radar design [10] where Latency Theory was the future-certainty dual of the past-uncertainty Information Theory. The third major theory discovered through POP guidance was in 2008 [2]. This third theory was the revelation of the laws of retention in physics, among which one finds the SDER, that was the past-uncertainty space dual of the future-certainty laws of motion in physics.

In the finding of the laws of retention in physics in [2] the POP first guided the search for the space dual of the speed of light in a vacuum which naturally led first to the investigation of extreme duality mediums for the best possible future-certainty motion of mass-energy and the best possible past-uncertainty retention of mass-energy. One of these mediums was a vacuum which yielded for motion problems the maximum speed of light energy in a vacuum with symbol c whose value was 2.9979 x 10^8 m/sec, while the other extreme medium was a black hole which yielded for retention problems the maximum pace of dark energy in a black hole with symbol χ whose value was 6.13 x10^{63} sec/m^3. The value for χ was found making use of the analytic expression $\chi = 960\pi c^2/Gh$, which was originally derived in [2] and whose derivation is also given in Appendix A of this paper where G is the gravitational constant and h is the Planck constant. After the finding of this dual for the speed of light, the path charted by Einstein for the solution to his relativity theory of motion can then be readily followed to find the SDER properties. In this paper we will only consider the most basic SDER properties that would lead us to dark energy predictions that are in perfect accord with the observations made by the team of astronomers in [3]- [4].

The remainder of the paper will be presented in two sections. In the first section the time-clock that powered the uncovering by Einstein of his relativity theory of motion (RTM) will be described first along with RTM predictions which are inside ‘the proper RTM domain’ as well as some predictions that have been made in the past that are outside the RTM domain and are thus invalid. In the second section the volume-clock, the space dual of the RTM’s time-clock, that powered the uncovering by the author of his SDER’s relativity theory of retention (RTR) will be described along with RTR predictions which are inside ‘the proper RTR domain.’ Predictions will include that of a vacuum inside the black hole where a novel quantum gravitational theory will be presented that would avert the gravitational collapse of the BH’s event horizon into a point. In the last section conclusions and further SDER predictions will be given.

2. Einstein’s Relativity of 1905

In Fig. 4a a moving time-clock with a fixed speed v is shown which is used to measure the always increasing passing of time in the formulation of Einstein’s relativity theory of motion. The speed v will constantly be less than the speed of light in a vacuum c whose value will always be measured to be the same regardless

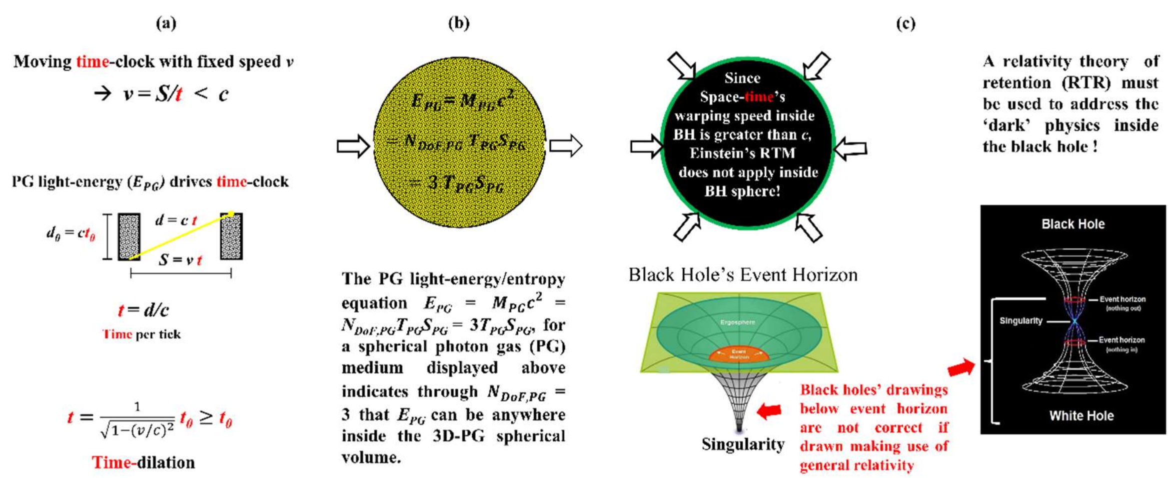

Fig. 4. (a) The moving time-clock with fixed speed v in Einstein’s relativity. (b) The photon gas light-energy E_{PG} driving the time-clock. (c) The black hole inside, where Einstein’s relativity does not work.

of the speed of the motion-frame of reference used. In Fig. 4a the time-tick of the moving time-clock that is measured by a motionless (stationary) time-clock will produce two motion relationships. The first relationship would be $t=d/c$ where 't' is the time-tick of the moving time-clock that is enabled by a photon gas (PG) light-energy E_{PG}, see Fig. 4b, that is in motion in a vacuum with speed c whose motion between two space-points within the moving time-clock results in the space-dislocation d as measured by the motionless time-clock. The second relationship would be $t=S/v$ where v is the fixed speed of the moving time-clock and S is the space change amount of the moving time-clock during the passing of the time t. A third relationship is for the time-tick of a motionless time-clock which yields $t_0=d_0/c$ where t_0 is the time-tick of the motionless time-clock also enabled by the PG light-energy E_{PG} that is in motion in a vacuum with speed c whose motion between two space-points within the motionless time-clock results in the space-dislocation d_0. Next after the three space-time relationships given by $t=d/c$, $t=S/v$, and $t_0=d_0/c$ are related through the Pythagorean space relationship stated as $(d=ct)^2 = (S=vt)^2 + (d_0 = ct_0)^2$, one arrives at the time-dilation expression $t=\gamma t_0$ where γ is the motion's Lorentz factor $1/\sqrt{1-(v/c)^2}$. The basic relativity of motion implication of the derived time-dilation equation can be readily illustrated using a pair of twins where one remains motionless, i.e., without any significant space dislocations while this twin's aging process continuous, i.e., the passing of time would continue to increase steadily, while the second twin changes from a motionless state to an inertial motion-frame of reference with a fixed speed v and then returns back to the original motionless state. After the second twin returns back to the original motionless state, he will find that he would be younger, i.e., the passing of time would have been less for him than the twin whose motionless state never changed.

The Einstein's relativity formulation presented so far only relates to Einstein's special relativity which he offered in 1905. Later in 1916 Einstein extended his special relativity theory to general relativity which deals with gravitation where he discovered that the existence of mass-energy in space-time results in the warping of space-time. This warping can be so intense that when created by a superdense mass it can generate warping speeds on space-time that can match the speed of light in a vacuum, which would then end the ability of light to move out of the influence of the warping mass-energy into free space. For instance, as depicted in Fig. 4c this would occur in a black hole scenario. As a result, the theoretical study of what goes on inside the black hole's event horizon using Einstein's relativity theory of motion is not physically as well as technically justified. In the next section it will be found however that the SDER offers a powerful as well as valid relativity theory of retention that avoids the misuse of Einstein's relativity in the study of the black hole's event horizon and beyond.

3. Space Dual of Einstein's Relativity of 2008

In this section the SDER will be discussed using exactly the same format as that used in Section 2 to review Einstein's relativity. This will make it significantly easier to become familiar with the new terminology that will be used in the description of the SDER.

In Fig. 5a a retaining volume-clock with a fixed pace Π is shown which is used to measure the always increasing passing of volume in the formulation of the SDER's relativity theory of retention. It is highlighted here that this 'increasing passing of volume that a volume-clock measures' is the SDER's space dual of the 'increasing passing of time that a time-clock measures' in Einstein's relativity. Moreover, it is noted that the claim by the team of astronomers that "supermassive black holes have grown so much that

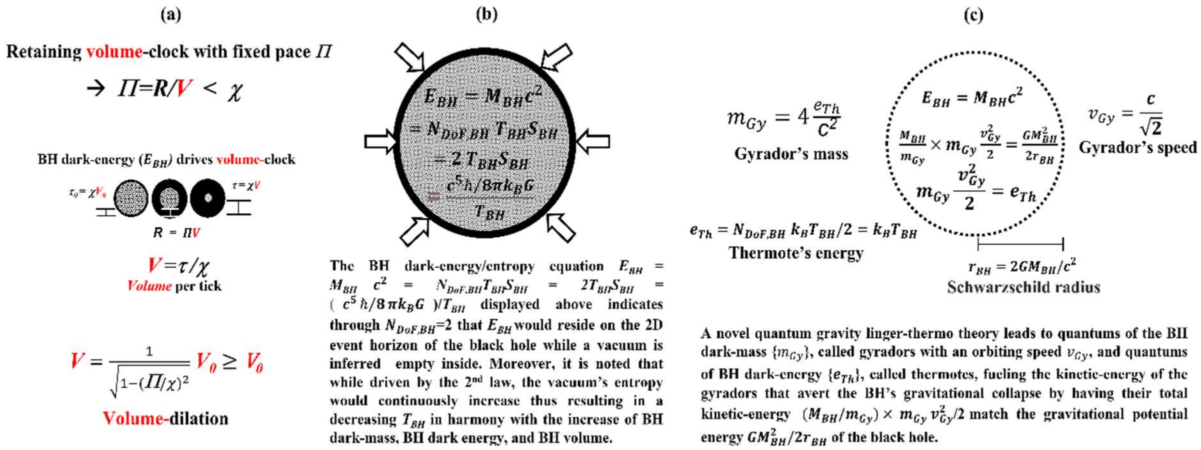

Fig. 5. (a) The retaining volume-clock with fixed pace *Π* of the SDER. (b) The BH dark-energy E_{BH} driving the volume-clock. (c) The gyrador mass quantum fueled by the thermote energy quantum that averts BH collapse.

they must be coupled to cosmic expansion" would require for theoretical support a framework like the SDER which requires a volume-clock for measuring the increasing passing of volume of the black hole.

Continuing with our duality description, the pace *Π* of the retaining volume-clock would always be less than the pace of dark in a black hole χ where the value of χ would be measured to be the same in any retention-frame of reference used according to the SDER duality. In Fig. 5a the volume-tick of the retaining volume-clock that is measured by a retentionless (stationary) volume-clock would produce two retention relationships. The first relationship would be $V=\tau/\chi$ where '*V*' is the volume-tick of the retaining volume-clock that is enabled by the BH dark-energy E_{BH}, see Fig. 5b, that is in retention in a black hole with pace χ whose retention between two linger-points within the retaining volume-clock results in the linger-dislocation τ as measured by the retentionless volume-clock. The second relationship would be $V=R/\Pi$ where *Π* is the fixed pace of the retaining volume-clock and *R* is the linger change amount of the retaining volume-clock during the passing of the volume *V*. A third relationship for the volume-tick of a retentionless volume-clock which yields $V_0=\tau_0/\chi$ where V_0 is the volume-tick of the retentionless volume-clock also enabled by the BH dark-energy E_{BH} that is in retention in a black-hole with pace χ whose retention between two linger-points within the retentionless volume-clock results in the linger-dislocation τ_0. Next after the three linger-volume relationships given by $V=\tau/\chi$, $V=R/\Pi$, and $V_0=\tau_0/\chi$ are related through the Pythagorean linger-volume relationship stated as $(\tau=\chi V)^2 = (R=\Pi V)^2 + (\tau_0=\chi V_0)^2$, one arrives at the volume-dilation expression $V=\xi V_0$ where ξ is the retention's Lorentz factor $1/\sqrt{1-(\Pi/\chi)^2}$. The basic relativity of retention implication of the derived volume-dilation expression $V=\xi V_0$ can be readily illustrated using a pair of identical black holes where one remains retentionless, i.e., without any significant linger dislocations while the increase of the volume of the black hole continuous, i.e., the passing of volume would continue to increase steadily, while the second black hole changes from a retentionless state to an inertial retention-frame of reference with a fixed pace *Π* and then returns back to the original retentionless state. After the second black hole returns back to the original retentionless state, the second black hole will be found to have a smaller volume, i.e., the passing of volume would have been less for this black hole, than the first black hole who's retentionless state never changed. Thus retentionless (isolated) black holes would have a greater volume growth rate than black holes that are actively retaining incoming outside materials and other black holes. Most significantly, regarding the accelerated expansion of the universe through dark-energy in black holes, the previously described volume-dilation would provide the theoretical mechanism for an accelerated expansion for the universe. More specifically, if we were to extend our analysis to black holes of the earlier universe, say less than 7 billion light years of age, one would find that the groups of black holes in a retentionless state, linked to significant volume growth, would be less likely to have been a dominating factor in the earlier universe expansion since the mass density of the universe would have been significantly greater thus making less likely the occurrence of isolated black holes. That is, in the earlier universe it was significantly more likely that black holes would not be in an isolated state, but rather they would be actively retaining close by materials and other black holes. On the other hand, from the time of 7 billion light years ago to now the mass density of the universe would have been found to be significantly less thus leading to isolated black holes leading to the accelerated expansion of the universe. As a result, it may be concluded that the SDER's volume-dilation does provide the theoretical foundation for the observed accelerated expansion of the universe through black holes!

Since 2008 when volume-dilation was first theoretically stated in [2], the energy source for the BH dark-energy E_{BH} has been directly related to the vacuum inside the black hole according to the SDER theory. To see this the black hole is noted to be spherical in shape as shown in Fig. 5b where the surface area is the event horizon, where anything outside it is treated by Einstein's relativity and anything inside by the SDER. The radius of the black hole would then control where Einstein's relativity applies and where the SDER does. This radius is given by the Schwarzschild radius r_{BH} according to:

$$r_{BH} = 2GM_{BH}/c^2 \tag{1}$$

where G is the gravitational constant and c is the escape speed from the black hole except that it cannot be achieved because the warping speed of space-time at the event horizon does not allow light to exit the black hole since it matches that of the speed of light. Next it is noted in Appendix B, first done in [11], that the Bekenstein Hawking entropy of the black hole can be expressed in terms of the number of degrees of freedom $N_{DoF,BH}$ of the black hole according to:

$$S_{BH} = k_B \frac{c^3}{\hbar G} \pi r_{BH}^2 = \frac{E_{BH}}{N_{DoF,BH} T_{BH}} = \frac{E_{BH}}{2T_{BH}} \tag{2}$$

where G is the gravitational constant and $\hbar$ is the reduced Planck constant. The black hole entropy expression (2) indicates through its explicit dependence on $N_{DoF,BH}=2$ that all activity takes place on the event horizon of the black hole. Thus, according to (2) the BH dark-energy E_{BH} and the BH temperature T_{BH} that generates the BH entropy S_{BH} occurs only on the black hole's 2D event horizon, a result suggesting the existence of a vacuum inside the black hole. Moreover, using in (2) the mass-energy expression $E_{BH} = M_{BH}c^2$ and the Schwarzschild radius equation (1) one would arrive then at the following expressions for the BH dark-mass and the BH dark-energy:

$$M_{BH} = (c^3\hbar/8\pi k_B G)/T_{BH} = 1.22 \times 10^{23} \ / \ T_{BH} \tag{3}$$

$$E_{BH} = (c^5\hbar/8\pi k_B G)/T_{BH} = 3.464 \times 10^{40} \ / \ T_{BH} \tag{4}$$

Equations (3) and (4) inform us, in addition, that the mass and energy of a black hole is inversely proportional to its temperature, thus as the BH temperature decreases the dark-mass and dark-energy of the black hole would increase. A vacuum, in turn, inside the black hole whose extremely small temperature would further decrease as the entropy of the universe increases due to the second law of thermodynamics could then act as a thermodynamics sink enabling the lowering of the BH temperature T_{BH} through heat flow with the passing of time. Thus, the SDER has been shown to support the observation by the astronomers of nine countries that dark energy exists in black holes and through volume-dilation work together to accelerate the expansion of the universe since around 7 billion light years ago.

The only thing that remains at this point is to provide a theory that would explain how the gravitational collapse of the black hole's event horizon to a point in space that would end the black hole's vacuum and thus the source of the growth of the BH dark-energy would not occur. The theory is a quantum gravity theory composed of mass and energy quantums that naturally emerged through the POP. The mass quantum is of the BH dark-mass M_{BH}, called gyrador with mass m_{Gy} that circles the spherical black hole's surface area (event horizon) with an orbiting speed v_{Gy}. The energy quantum is of the BH dark-energy E_{BH}, called thermote with energy e_{Th} which is the thermal energy contributed by the number of degrees of freedom $N_{DoF,BH}$ for BH particle motions given according to:

$$e_{Th} = N_{DoF,BH} k_B T_{BH}/2 = k_B T_{BH}. \qquad (5)$$

The task of the thermote is then to fuel the kinetic-energy of the gyrador according to:

$$m_{Gy} v_{Gy}^2/2 = e_{Th} = k_B T_{BH} \qquad (6)$$

thus, giving rise to the following expression for the gyrador mass:

$$m_{Gy} = 2k_B T_{BH}/v_{Gy}^2 \qquad (7)$$

The orbiting speed of the gyrador v_{Gy} would then be found that prevents the gravitational collapse of the black hole's event horizon of the gyrador by setting the total kinetic-energy of M_{BH}/m_{Gy} gyradors equal to the gravitational potential energy $GM_{BH}/2r_{BH}$ of the black hole according to:

$$(M_{BH}/m_{Gy})\, m_{Gy} v_{Gy}^2/2 = GM_{BH}/2r_{BH} \qquad (8)$$

Making use of (8) one then arrives at the following expression for the orbiting speed of the gyrador:

$$v_{Gy} = (GM_{BH}/r_{BH})^{1/2} = c/\sqrt{2} \qquad (9)$$

where use has been made in (9) of Schwarzschild radius (1). Finally making use of (9) in (7) one then arrives at the following expression for the mass of the BH gyrador:

$$m_{Gy} = 4k_B T_{BH}/c^2\,. \qquad (10)$$

where it is of interest to note the gyrador mass and thermote energy are both linear function of the BH temperature while for the case of the BH mass and BH energy they are both inversely proportional to T_{BH}. Thus, it has been shown that the gravitational collapse of the black hole's event horizon to a point is theoretically averted by orbiting gyrador mass quantums at around 71% of the speed of light (9) that are fueled by thermote energy quantums whose value is proportional to T_{BH}. It is finally noted that the quantum gravity theory used here to avert gravitational collapse has been used in the patent application [12] to control the temperature of chemical reactions in a non-equilibrium state using a mass balance rather than a thermocouple. Moreover, the quantum gravity theory has provided theoretical support for a visible universe, not one filled with dark-matter, see iPoster for the 2020 AAS 235 Winter Meeting [16].

4. Conclusions

In this paper the Space Dual of Einstein's Relativity (SDER) first reported in a SPIE article fifteen years ago has been revisited while further advancements have also been reported. Four basic theoretical results were reviewed for the SDER. They were:

1) The event horizon of the black hole through the Schwarzschild radius provided a natural theoretical and physical boundary for either the use of Einstein's relativity or the use of the SDER from our outside observer's perspective. This outside perspective constrained us to only being able to gather provable information about what occurred on the black hole's event horizon, not its inside which cannot be monitored.

2) The SDER's pace of dark in a black hole χ was established as the space dual of Einstein's relativity's speed of light in a vacuum c which provided an upper limit for the pace of retention in any medium just like c does for the speed of motion in any medium.

3) The SDER's volume-clock driven by BH dark energy for the measurement of the always increasing passing of volume was established as the space dual of Einstein's time-clock driven by PG light-energy for the measurement of the always increasing passing of time.

4) The SDER's retaining volume-clock with BH dark-energy in a retention-frame of reference at a constant pace was established as the space dual of Einstein's relativity's moving time-clock with PG light-energy in a motion-frame of reference at a constant speed.

The above four theoretical duality results established the SDER as requiring the continuous increase of the black hole's volume. It also provided for a retaining black-hole that was ruled by a retention-frame of reference with a constant pace that resulted in a volume increase that was lower than that of a retentionless (isolated) black hole that was ruled by a retentionless volume-clock. These two theoretical results when viewed from the universe's perspective of an early expansion and a later accelerated expansion of the universe via supermassive black holes in isolation were fully supportive of the early expansion and the later accelerated expansion of the universe.

Next to establish the source of the energy that would be consistent with the established SDER theoretical results, the Bekenstein Hawking entropy for the black hole was used to produce the following two fundamental and sensible theoretical results:

1) The BH dark-mass and the BH dark-energy were both found to be inversely proportional to the BH temperature whose value is often extremely small.

2) Inside the black hole the finding of a vacuum was suggested by the form of the BH entropy result which was directly related to the two degrees of freedom of the event horizon. This result suggested no activity inside the black hole at all that would have impacted its entropy. Thus, consistent with the appearance inside the black hole of a vacuum whose temperature was very close to zero but never reaching it due to the third law of thermodynamics, and further decreasing as the universe increased its entropy due to the second law of thermodynamics.

The above two theoretical BH entropy results then provided the necessary increase in the BH dark-energy by having the vacuum temperature acting as the thermodynamics sink where heat would flow from the decreasing BH temperature, which in turn increased the BH dark-energy as required. Thus, theoretically revealing for us the actual source of the sought after dark-energy!

Finally, a novel quantum gravity theory was used to theoretically explain how the gravitational collapse of the black hole's event horizon could be averted. This was done by introducing gyradors, quantums of the BH dark-mass, orbiting the event horizon at around 71% of the speed of light whose kinetic energy was fueled by thermotes, quantums of the BH dark-energy. The total kinetic energy of all the gyradors was then used to match the gravitational potential energy of the black hole, thus avoiding the gravitational collapse of the black hole's event horizon. This gravitational collapse result would then infer that everything entering a black hole would remain in the event horizon vicinity rotating at extremely high speeds, another SDER prediction!

The previously summarized SDER theoretical results have been found to be perfectly supportive of the observations reported by astronomers from nine countries of the existence of dark energy in black holes that would be responsible for the universe's accelerated expansion. It is expected that future investigations of the SDER's relativity theory of retention would lead us to other sensible predictions in astrophysics and other related fields. For instance, since a volume-clock is the space dual of a time-clock, a sensible prediction would then be that like there is an acceleration in the creation of the universe's volume there would be one for the passing of time.

Appendix A

The Pace of Dark in a Black Hole χ

In this appendix the maximum pace of energy retention in any medium, which occurs in an uncharged, and non-rotating black hole, is derived. This maximum pace value was first derived in [2] where it was given the name "pace of dark in a black hole." Its assigned symbol since then has been χ and the expression from which its value arise that will be derived next given according to:

$$\chi = \frac{\tau}{V} = \frac{480c^2}{\hbar\, G} = 6.13 \text{ x } 10^{63}\ sec/m^3 \quad \text{(A.1)}$$

where τ is the lifespan of the medium's energy in the black hole whose volume is V, c is the speed of light in a vacuum, $\hbar$ is the reduced Planck constant and G is the gravitational constant. The investigated black hole is spherical in shape with this volume given by the expression:

$$V = 4\pi r_{BH}^3 / 3 \tag{A.2}$$

where its radius r_{BH} is given by the Schwarzschild radius expression:

$$r_{BH} = \frac{2GM}{c^2}. \tag{A.3}$$

The derivation starts with the following black body luminance power expression [14] assumed to hold for the spherical in shape black hole:

$$-\frac{dE}{dt} = \frac{\pi^2}{60\hbar^3 c^2}(4\pi r_{BH}^2)(k_B T)^4 \tag{A.4}$$

where k_B is the Boltzmann constant, T is the driving heat temperature of the black hole and $4\pi r_{BH}^2$ is the black hole's surface area. Next the Schwarzschild radius for a black hole (A.3) together with the black hole's mass-energy E=Mc^2 are used in the Bekenstein-Hawking (BH) entropy [15] expression for a black hole given by:

$$S_{BH} = k_B \frac{c^3}{\hbar G}\pi r_{BH}^2 \tag{A.5}$$

to yield:

$$S_{BH} = k_B \frac{4\pi G}{\hbar c^5} E^2. \tag{A.6}$$

The thermodynamics temperature expression $T = (\partial S_{BH} / \partial E)^{-1}$ is then applied to equation (A.6) to derive:

$$\frac{1}{T} = k_B 2\frac{4\pi G}{\hbar c^5} E \tag{A.7}$$

Next making use of equation (A.7) for the temperature T in (A.4), as well as using in (A.4) the radius r_{BH} expression (A.3) with M=E/c^2, one then obtains the following luminance's non-linear differential equation in E:

$$-\frac{dE}{dt} = \frac{\hbar c^{10}}{15360\,\pi G^2 E^2} \tag{A.8}$$

Solving equation (A.8) one then finds the following time-varying $E(t)$ solution:

$$E^3(t) = E^3(t)\Big|_{t=0} - \frac{\hbar c^{10}}{5120\pi G^2}t = E^3 - \frac{\hbar c^{10}}{5120\pi G^2}t \tag{A.9}$$

where E is the value of $E(t)$ when the t is set equal to the assumed initial time of zero. Next the lifespan of operation of the black hole τ is found by setting $E(t)$ equal to zero which then yields:

$$t\big|_{E(t)=0} = \tau = \frac{5120\pi G^2}{\hbar c^{10}} E^3 . \tag{A.10}$$

Next equation (A.3) with M replaced with E/c^2 is used to arrive at

$$E = r_{BH} \frac{c^4}{2G} . \tag{A.11}$$

Finally using for the black hole the energy E expression (A.11) and also the volume V expression (A.2) in the lifespan τ equation (A.10), the desired result pace of dark result (A.1) is revealed.

Appendix B

The Black hole and Photon Gas Thermote-Entropy Equations

In this appendix, the known entropies of a black hole and a photon gas will be solely expressed in terms of their number of degree of freedom N_{DoF}, temperature T, and energy E. For a photon gas within a spherical volume its photons would be able to move in three-dimensional space while for a black hole whose volume is also spherical, its particles would be able to only move in the two-dimensional space of its event horizon, thus implying that there is not particle activity inside the black hole except for pairs of virtual particles created in the vacuum within. We next derive the number of degrees of freedom (DoF) based entropies of the black hole and the photon gas mediums first derived in [11].

B.1. The DoF Black Hole Entropy

In this subsection it is shown that the Bekenstein Hawking entropy of an uncharged, non-rotating black hole (BH) S_{BH} [15] can be simply expressed [11] in terms of the BH's number of degrees of freedom $N_{DoF,BH}$, which is equal to two, the BH temperature T_{BH}, and the BH dark energy $E_{BH}=M_{BH}c^2$ where M_{BH} is the BH dark mass according to:

$$S_{BH} = \frac{E_{BH}}{N_{DoF,BH}T_{BH}} = \frac{E_{BH}}{2T_{BH}} \tag{B.1}$$

or alternatively by expressing the BH dark energy E_{BH} in terms of $N_{DoF,BH}$, T_{BH}, and S_{BH} as follows:

$$E_{BH} = N_{DoF,BH}T_{BH}S_{BH} = 2\,T_{BH}S_{BH}. \tag{B.2}$$

The derivation of (B.1) begins with the Bekenstein-Hawking entropy expression for the uncharged, non-rotating black hole which is also spherical in shape:

$$S_{BH} = k_B \frac{c^3}{G\hbar}\pi r_{BH}^2 = k_B \frac{c^3}{G\hbar}\pi r_{BH}^{N_{DoF,BH}} \tag{B.3}$$

where $N_{DoF,BH}$=2 indicates the two physical dimensions associated with the event horizon of the black hole, c is the speed of light, G is the gravitational constant, $\hbar$ is the reduced Planck constant, and r_{BH} is the radius of the black hole whose value is given by the Schwarzschild radius expression:

$$r_{BH} = 2GM_{BH}/c^2. \tag{B.4}$$

The BH dark energy expression E_{BH}=$M_{BH}c^2$ is then used in (B.4) with its result then placed in (B.3) to derive the BH entropy expression:

$$S_{BH} = k_B \frac{\pi G\, 2^{N_{DoF,BH}}}{\hbar c^{-3+4N_{DoF,BH}}} E_{BH}^{N_{DoF,BH}}. \tag{B.5}$$

Next making use of (B.5) in the thermodynamics definition for temperature in terms of entropy $T_{BH} = (\partial S_{BH}/\partial E_{BH})^{-1}$ [14] the following relationship between temperature and energy emerges:

$$\frac{1}{T_{BH}} = k_B \frac{\pi G\, 2^{N_{DoF,BH}}}{\hbar c^{-3+4N_{DoF,BH}}} N_{DoF,BH} E_{BH}^{N_{DoF,BH}-1}. \tag{B.6}$$

After combining (B.6) and (B.5) the desired result (B.1) finally emerges.

B.2. The DoF Photon Gas Entropy

In this subsection it is shown that the entropy of a photon gas S_{PG} [14] can be simply expressed in terms of the PG's number of degrees of freedom $N_{DoF,PG}$, which is equal to three, the PG temperature T_{PG}, and the PG light energy E_{PG}=$M_{PG}c^2$ where M_{PG} is the PG light mass according to:

$$S_{PG} = \frac{E_{PG}}{N_{DoF,PG}T_{PG}} = \frac{E_{PG}}{3T_{PG}} \tag{B.7}$$

or alternatively by expressing the PG light energy E_{PG} in terms of $N_{DoF,PG}$, T_{PG}, and S_{PG} as follows:

$$E_{PG} = N_{DoF,PG}T_{PG}S_{PG} = 3\,T_{PG}S_{PG}. \tag{B.8}$$

The derivation of (B.7) begins with the known photon-gas entropy expression [14] given by:

$$S_{PG} = k_B \frac{4\pi^2 (k_B T_{PG})^3 V_{PG}}{45\ \ {}^3\hbar^3} \tag{B.9}$$

where the volume V_{PG} of the photon gas, assumed spherical, is given by the expression:

$$V_{PG} = 4\pi \frac{r_{PG}^3}{3} = 4\pi \frac{r_{PG}^{N_{DoF,PG}}}{3} \tag{B.10}$$

where $N_{DoF,PG}$=3 indicates the three physical dimensions associated with the photon gas volume with its radius r_{PG} related to the escape speed from the photon gas according to:

$$v_e^2 = \frac{2GM_{PG}}{r_{PG}} \tag{B.11}$$

The PG light-energy E_{PG}=$M_{PG}c^2$ is then used in (B.11) with its result then placed in (B.10) whose result, in turn, in used in (B.9) to derive the PG entropy expression:

$$S_{PG} = k_B \frac{128\pi^3 G^3 (k_B T_{PG})^3}{135 c^9 \hbar^3 v_e^6} E_{PG}^{N_{DoF,PG}} \quad \text{(B.12)}$$

Then applying the thermodynamics temperature equation $T_{PG} = (\partial S_{PG}/\partial E_{PG})^{-1}$ [14] to (B.12) one finds:

$$\frac{1}{T_{PG}} = k_B \frac{128\ {}^3 G^3 (k_B T_{PG})^3}{135 c^9 \hbar^3 v_e^6} N_{DoF,PG} E_{PG}^{N_{DoF,PG}-1} \quad \text{(B.13)}$$

After combining (B.13) and (B.12), the desired result (B.7) finally emerges.

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